

## CONTENTS

<i>Preface</i>	<i>ix</i>
<i>Introduction</i>	1
<b>1 Test functions and distributions</b>	4
1.1 Some notations and definitions	4
1.2 Test functions	5
1.3 Distributions	7
1.4 Localization	10
1.5 Convergence of distributions	13
Exercises	15
<b>2 Differentiation, and multiplication by smooth functions</b>	17
2.1 The derivatives of a distribution	17
2.2 Some examples	18
2.3 A distribution obtained by analytic continuation	20
2.4 Primitives in $\mathcal{D}'(\mathbf{R})$	22
2.5 Product of a distribution and a smooth function	23
2.6 Linear differential operators	25
2.7 Division in $\mathcal{D}'(\mathbf{R})$	27
2.8 Duality	29
Exercises	30
<b>3 Distributions with compact support</b>	34
3.1 Continuous linear forms on $C^\infty(X)$ , and distributions with compact support	34
3.2 Distributions supported at the origin	36
Exercises	39
<b>4 Tensor products</b>	40
4.1 Test functions which depend on a parameter	40
4.2 Affine transformations	42
4.3 The tensor product of distributions	44
Exercises	48

<i>Contents</i>	<i>vi</i>
<b>5 Convolution</b>	<b>50</b>
5.1 The convolution of two distributions	50
5.2 Regularization	53
5.3 Convolution of distributions with non-compact supports	55
5.4 Fundamental solutions of some differential operators	59
Exercises	65
<b>6 Distribution kernels</b>	<b>68</b>
6.1 Schwartz kernels and the kernel theorem	68
6.2 Regular kernels	73
6.3 Fundamental kernels of differential operators	76
Exercises	78
<b>7 Coordinate transformations and pullbacks</b>	<b>80</b>
7.1 Diffeomorphisms	80
7.2 The pullback of a distribution by a function	81
7.3 The wave equation on $\mathbf{R}^4$	85
Exercises	88
<b>8 Tempered distributions and Fourier transforms</b>	<b>90</b>
8.1 Introduction	90
8.2 Rapidly decreasing test functions	93
8.3 Tempered distributions	96
8.4 The convolution theorem	101
8.5 Poisson's summation formula, and periodic distributions	104
8.6 The elliptic regularity theorem	108
Exercises	110
<b>9 Plancherel's theorem, and Sobolev spaces</b>	<b>114</b>
9.1 Hilbert space	114
9.2 The Fourier transform on $L_2(\mathbf{R}^n)$	116
9.3 Sobolev spaces	120
Exercises	126
<b>10 The Fourier-Laplace transform</b>	<b>128</b>
10.1 Analytic functions of several complex variables	128
10.2 The Paley-Wiener-Schwartz theorem	130
10.3 An application to evolution operators	134
10.4 The Malgrange-Ehrenpreis theorem	139
Exercises	142
<b>11 The calculus of wavefront sets</b>	<b>144</b>
11.1 Definitions	144
11.2 Transformations of wavefront sets under elementary operations	148
11.3 Push-forwards and pull-backs	154
11.4 Wavefront sets and Schwartz kernels	157
11.5 Propagation of singularities	159
Exercises	160

<i>Contents</i>	<i>vii</i>
<i>Appendix: topological vector spaces</i>	162
<i>Bibliography</i>	170
<i>Notation</i>	171
<i>Index</i>	173