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# *Introduction to the theory of distributions*

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with additional material by

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**CAMBRIDGE**  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

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It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521640152](http://www.cambridge.org/9780521640152)

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First published 1982

Second Edition 1998

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-64015-2 Hardback

ISBN 978-0-521-64971-1 Paperback

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## PREFACE

The aim of this book is to give a straightforward, and largely self-contained, introduction to the theory of distributions. It is based on lectures given in Cambridge over a number of years, to audiences who could not be expected to have much familiarity with such prerequisites as functional analysis, or Lebesgue integration. Distributions are therefore defined by means of seminorm estimates; the relation of these to the underlying topological vector space theory is sketched in an appendix. Standard theorems on integrals which are needed are stated without proof. Such a presentation entails some loss, but this is offset, one hopes, by its wider accessibility. An interested reader can deepen his or her understanding by further study.

The first eight chapters of the book provide a basic course on the subject. The last two, which are independent of each other, carry the theory of Fourier transforms of distributions a little further. The theory is developed from the outset on open sets of  $\mathbf{R}^n$ , as the field in which distributions have been applied most conspicuously and fruitfully is the theory of partial differential equations. Some of these applications are included, often as exercises.

My debt to the literature, and in particular to L. Schwartz's treatise, and L. Hörmander's account of the subject in the first chapter of his *Linear Partial Differential Operators*, is apparent. In addition, I have been able to consult a set of unpublished lecture notes by Professor Hörmander. I also want to thank Richard Melrose, who has read the book in manuscript, and at the same time absolve him from any errors, for which I must take sole responsibility.

## PREFACE TO THE SECOND EDITION

For this edition, a section on wave front sets has been added. This has been written by Dr Mark Joshi, and I want to thank him for his contribution. I have also used the opportunity to make some minor revisions and corrections. I am grateful to Ben Fairfax and Aurelian Bejancu, who have provided me with a comprehensive list of misprints, and raised some queries that have, I hope, led to improvements in the text.

Gerard Friedlander