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0521637694 - New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria, and Externalities as Field Effects

Masanao Aoki

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This book contributes substantively to the current state of art of macroeconomic modeling by providing a method for modeling large collections of possibly heterogeneous agents subject to nonpairwise externality called field effects, that is, feedback of aggregate effects on individual agents or agents using state-dependent strategies. By adopting a level of microeconomic description that keeps track of compositions of fractions of agents by types or strategies, time evolution of the microeconomic states is described by (backward) Chapman–Kolmogorov equations. Macroeconomic dynamics naturally arise from these equations by expansion of the solutions in some power series of the number of participants. Specification of the microeconomic transition rates thus leads to macroeconomic dynamic models. This approach provides a consistent way for dealing with multiple equilibria of macroeconomic dynamics by ergodic decomposition and associated calculations of mean first passage times, and stationary probabilities of equilibria provide further useful information on macroeconomic behavior.

This book ends with a set of elaborations, sketches of further topics of research, and a collection of supporting materials in the Appendix.

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# **New Approaches to Macroeconomic Modeling**

**Evolutionary Stochastic Dynamics, Multiple  
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MASANAO AOKI

*University of California, Los Angeles*



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*To My Mother*

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## PREFACE

This book is an attempt to reformulate macroeconomic modeling as currently practiced by the macroeconomic profession.

The need to improve macroeconomic models certainly is felt widely by the economic profession. A short list of the defects that we recognize in macroeconomic modeling includes extensive and almost exclusive use of the assumption of representative agents, of largely deterministic dynamic models, and inadequate attention paid to off-equilibrium dynamic phenomena. More specifically, we do not have a satisfactory model for explaining sluggish responses of macroeconomic phenomena and the distributional effects of policy actions. We lack adequate treatments of the dynamic adjustment behavior of microeconomic units in the presence of externalities of the kind designated “field effects” in this book, and are known variously in the economic literature as social influence, social consumption, or group sentiments or pressure.

This book collects my recent investigations to provide an alternative manner for building and analyzing models in macroeconomics; it is addressed to macroeconomists and advanced graduate students in macroeconomics. The book is arranged in three parts. Part I consists of three chapters. After a short introductory discussion of motivation for developing a new way to construct and analyze macroeconomic models in Chapter 1, Chapter 2 provides some simple, motivating examples of the proposed approaches. An explicitly stochastic or statistical approach is taken in Chapter 3. It collects some material that I use in the remainder of the book, since this material is not usually in the toolkit of practicing macroeconomists and is not taught in traditional economics graduate courses.

Part II, consisting of Chapters 4 through 6, presents models of interacting microeconomic units via jump Markov processes and the derivation of macroeconomic dynamics by backward Chapman–Kolmogorov equations. Time evolution of a large collection of microeconomic units is described in terms of probability densities, and is governed by the master equation. Under certain

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conditions, the equilibrium probability distributions are Gibbs distributions. Chapter 4 addresses situations in which each of the microeconomic units has a finite number of choices, and thus extends the literature on discrete choice to include interacting agents.

Chapter 5 is a further elaboration of the ideas presented in Chapter 4. It focuses on the type of externalities that I call field effects. Field effects are such that each agent is influenced by the aggregate effects, and hence are usually weak and diffuse. These effects are in contrast with the pairwise interactions between two, possibly anonymous, agents. In Chapter 5 as well as in Chapter 4, a new way of aggregating microeconomic units to produce macroeconomic equations is illustrated. The issue associated with the existence of multiple macroeconomic equilibria naturally arises. In the explicitly stochastic framework of this book, dynamic models with well-behaved basins of attractions for locally stable equilibria behave in such a way that each basin will be visited with a positive probability, and the expected time of transition from one locally stable equilibrium to another is shown to depend on the height of a potential barrier separating the two basins. Finally, pairwise interactions are taken up in Chapter 6.

Part III, composed of Chapters 7 and 8, examines properties of dynamics with a large state space, which are organized hierarchically. The characteristic feature of such dynamics is sluggishness in the dynamic responses. Once disturbed off an equilibrium dynamic path, a hierarchical state-space model returns to it at rates slower than the usual exponential ones exhibited by dynamics with state spaces that are not similarly organized. Chapter 8 treats gathering of agents into the same state (or beliefs), as in bubbles and other critical phenomena. The book ends with a collection of supporting materials in the Appendix.

Some of the topics have been presented in my graduate course on economic modeling at the University of California, Los Angeles, as well as in special courses or sequences of seminars that I was invited to present in the past three years at the University of Tokyo, the Åbo Academy University, the European University Institute, the University of Siena, and elsewhere. I thank my former students as well as seminar participants for their reactions and comments. I thank H. Yoshikawa, K. G. Nishimura, L. Punzo, R. Fiorito, M. Salmon, and R. Östermark for arranging such special seminars or courses.

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