CHAPTER 1

Introduction

The macroeconomic profession is well aware of the shortcomings of the current state of macroeconomic modeling, and is generally dissatisfied with the models it uses, as documented, for example, in Kirman (1992b) or Leijonhufvud (1993, 1995). The need to improve macroeconomic models is certainly felt widely. To cite a few examples, we do not have a satisfactory explanation of why some macroeconomic variables move sluggishly, or how policy actions affect segments of the economy differently, and we lack adequate tools for treating dynamic adjustment behavior of microeconomic units in the presence of externalities of the kind termed "social influence" by Becker (1990) and others, or in models with multiple equilibria.

Leijohnhufvud (1995) argues that the Rationality Postulate has such a strong hold on economists that, although they are aware of the notion of the Invisible Hand, a self-regulating order emerging in a complex system from the interactions of many microeconomic agents without this being part of their intentions, they are generally blind to hierarchical complexity and some of its consequences and averse to learning from other fields in which emergent properties and distributive information processing are also studied.

This book is concerned with modeling a large collection of not necessarily homogeneous microeconomic agents or units in a stochastic and dynamic framework. Major ingredients that distinguish this book from other books on macroeconomics are the following: (a) use of jump Markov processes to model interacting microeconomic agents; (b) focus on the multiplicity of microeconomic states that are consistent with given sets of macroeconomic observables, and introduction of exchangeable agents in addition to the usual distinguishable agents, since they affect the multiplicity counts; (c) a new type of analysis of multiple equilibria and calculation of mean first-passage times between equilibria, and introduction of the notion of ergodic components to describe state spaces with several basins of attraction; and (d) introduction of hierarchical structures to explain complex and sluggish dynamic responses of some macroeconomic variables in large dimensional state space. We take up some of these points next.

1.1 Multiplicity

This book discusses models composed of a large number of microeconomic agents. That a large number of microeconomic agents interact in our model has several important consequences. The most important one is undoubtedly the fact that a large number of microeconomic states are compatible with a given macroeconomic state. We call this the multiplicity of microeconomic states or the degeneracy of macroeconomic state. Although this fact is completely ignored in the existing macroeconomic literature, it has profound effects on the behavior of the macroeconomic models that we construct by aggregating models of microeconomic agents, because the multiplicity determines how uncertainty in the model affects macroeconomic behavior. In addition to the usual distributional effects of changes in macroeconomic states affecting segments of the economy differently, degeneracy affects equilibirum probability distribution of macroeconomic states. The probability distributions of microeconomic states consistent with a set of macroeconomic variables or observables are obtained by applying the Gibbs conditioning principle in Chapter 3.[1]

This is the reason why we discuss entropy and Gibbs distributions in Chapters 2 and 3. As we make clear there, entropy is a measure of the degree of multiplicity of microeconomic states, that is, degeneracy of a macroeconomic state. This point becomes more transparent when we discuss the exponents of the equilibrium probability distribution of the exponential type later in Chapter 5, where we observe that a given performance or cost index must be modified by a measure of the degeneracy of the optimal solution, which is measured by entropy.

Put differently, degeneracy is relevant to us because we take the view that the state space of an economy possesses basins of attraction that are associated with macroeconomic equilibria and, around them, states of the economy randomly move about, occasionally jumping from one basin to another. An economy may stay in this new basin for a while until it jumps out again. A basin is composed of microeconomic configurations that are compatible with or converge to the given macroeconomic state. We also can calculate the average time for the system to travel between two local equilibria, and show that the mean travel time will depend on the difference in the potentials[2] of the local equilibria.

[1] Fragments of this procedure are found in random utility models or random coefficient choice models, but these models are static rather than dynamic.

[2] For now, think of potentials as the exponents in exponential distributions.

1.2 Multiple equilibria

Multiple equilibria are likely to be present in dynamic models that have a large number of microeconomic agents. Multiple equilibria bring out the essential difference between deterministic and stochastic formulations of dynamic models. Suppose that a deterministic dynamic macroeconomic model has multiple equilibria with well-defined basins of attraction for each locally stable equilibrium. One important and natural question to pose for such a model is how the model can settle on an economically desirable equilibrium. With deterministic models, the only way to reach the desired equilibrium is to be initially in the basin of attraction associated with that equilibrium. Otherwise, the models must jump to the right basin. This is the issue of history versus perfect-foresight expectations addressed by Krugman (1991).

Unlike deterministic models, multiple equilibria present no conceptual difficulty to stochastic models. In the stochastic formulation in this book, we show that there are positive steady-state or stationary probabilities that the model state is in each basin, and the state moves from one equilibrium to another with positive probabilities, and mean first-passage times between these equilibria can, in principle, be calculated. The essence of this situation is captured by the example in Chapter 2, where the expected first-passage time from one equilibrium to another has been shown to depend exponentially on the height of the barrier separating the two basins, even though the equilibrium probabilities are independent of the height. Use of probabilistic or stochastic concepts and models allows us to regard macroeconomic regularities as statistical laws, and we employ empirical distributions and large deviation techniques to state statistical findings. Instead of a single ergodic system, we have ergodic decompositions in general. Each locally stable equilibrium has associated with it an ergodic component. There are positive probabilities associated with each ergodic component, and there are positive probabilities of transitions from one basin of attraction to another. Some of the mean first-passage times could be very long, indicating sluggishness of responses of macroeconomic phenomena.

The importance of and need to (re)cast choice or optimization problems of microeconomic units in a stochastic framework have been hinted at. Briefly, such reformulations involve three ingredients: spaces of states or strategies (decisions), which are called state spaces; transition rates that specify rates of changes of probabilities of transitions from one microeconomic state to another; and the deterministic equation for time evolution of the probabilities of states.

Long ago, Bellman (1961), pointed out that the state of a dynamic system (composed of microeconomic agents) is the probability distribution of (their microeconomic) states, because knowledge of the distribution determines uniquely probabilistically how the system evolves with time in the future. In this extended sense, state equations are deterministic. Unlike the evolutionary game literature, however, it is not the strategy mixes themselves that evolve deterministically. It is the probability distribution of the strategy mixes that evolves deterministically.[3] Because we choose microeconomic states to be discrete, we deal directly with sets of probabilities of the states, which we also call distributions. The so-called replicator dynamics in the literature of evolutionary games or Malthusian dynamics of Friedman are replaced with the backward Chapman–Kolmogorov equation. The fitness function of Friedman is replaced with transition rates that contain some functions, like the fitness function, that affect the rates. When transition rates satisfy the detailed balance conditions, the equilibrium distributions in the form of the Gibbs distribution exist, as discussed by Kelly (1979). These matters are discussed in Chapters 3 and 4.

1.3 Jump Markov processes

Next, we explain why we use jump Markov processes to model the behavior of microeconomic agents. Economic literature introduces many examples that show that (optimal) adjustment behavior by microeconomic units is not always continuous or small. Rather, adjustments are made at some discrete points in time by some finite magnitudes. Similar adjustment behavior is known to be optimal or used as convenient suboptimal rules in several classes of problems in finance, operations research, and control. Many of these reported results show that adjustment or decision (control) rules are of the threshold type: A specific action or decision is chosen or triggered when some gap variable, which measures gap or discrepancy between a desired or ideal value and the actual one, reaches or exceeds (becomes larger or smaller than) some preset threshold or trigger level. Adjustments could be in one direction only or they could be bidirectional, that is, adjustments could be upward or downward. More generally, when state variables of a microeconomic unit are in a certain subset of states, and when the unit receives a specific shock or signals from other units, then the decision of the unit is to choose a particular adjustment from a discrete-choice set. Microeconomic agents following these decision rules then act or adjust intermittently. Although the variable that represents the gap lies between the thresholds, no action or adjustment is undertaken. Adjustments of labor forces by firms and decisions of entry and exit by firms are also of this type.

In the economics literature, we find many examples of aggregate variables affecting microeconomic decisions, although they are not called field effects. Becker (1974, 1990), for example, examines an individual consumer's demand for some goods that depends on the demands by other customers, such as de-

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[3] See the survey of Friedman (1991) on evolutionary games.

mands for popular restaurant seats, theater tickets, and best sellers. What Becker calls the effects of social consumption is an example of what we call field effects, and is further discussed in Section 5.6. In the phenomena examined by Becker, there is no reason to think of interactions among agents as pairwise interactions, anonymous or otherwise. Caballero and Lyons (1990, 1992) also discuss the effects of aggregate output on sectoral or other country outputs. Becker (1974), Schelling (1978), Akerlof (1980), Friedman (1993), Friedman and Fung (1994), and Aoki (1995a) are also examples of field effects.[4] It is important to recognize that our method deals with interactions over time without the curse of dimensionality of the dynamic programming approach commonly used in the discrete-choice literature. The reader should note that none of the examples mentioned above, which are drawn from economic and related literature, analyzes interaction among microeconomic units.[5] At any given point in time, microeconomic agents have chosen some decisions, typically from finite-choice sets. Joint effects of their choices affect the economic environments in which they perform and make further choices in the future, that is, there is feedback from aggregate decisions to individual choices. Therefore, the agents operate in endogenous stochastic and dynamic environments, or they are subject to stochastic and dynamic externality. Effectiveness, desirability, or utility of their decisions is affected by the joint effects of other agents' actions, which determine the state variables of the models. We discuss interactions among microeconomic agents in Chapters 4 through 7.

Jump Markov processes provide an appropriate framework for modeling a class of models known as market participation models [see Stoker (1993)] or another class of models in which agents choose types interactively. There can be several types of microeconomic agents that interact in a model: optimizers and imitators, informed and uninformed, purchasers and nonpurchasers of information, investers and noninvestors, or holders of one kind of assets or another. Possibilities are many. Furthermore, random encounters of microeconomic agents of several types also can be modeled this way advantageously. Clearly, agents can choose their roles or types in addition to adjusting gap variables as part of their strategy or decision processes, and the problem can be described as a large collection of interacting jump Markov processes, or a birth-and-death process with state-dependent and possibly nonlinear transition rates.

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[4] It is interesting to read in Friedman (1991) that biologists use the expression "playing the field" to indicate interactions with a whole population. According to him, background environments and prevalence of alternative behavior traits or patterns in a population affects fitness of a particular biological trait or behavioral pattern in biological models.

[5] For example, as we discuss later, Caballero (1992), and Caballero and Engel (1992) use the Glivenko–Cantelli lemma to deal with the cross-sectional distribution of firms by discrete choices that they make. This theorem requires that firms' states are independent random variables.

We propose to model all these and other related problems that involve discrete choices made at intermittent time points as interacting jump Markov processes. Population compositions or mixes of agents with alternative decisions will evolve over time according to jump Markov processes. Specifications of decision processes for microeconomic agents determine probabilistic or stochastic mechanisms for the agents to change their decisions over time. These specifications are then translated into those of transition rates between microeconomic states. Transition rates between microeconomic states are generally easier to construct than direct specifications of macroeconomic models.

Our approach is easier to apply, and generally more powerful in circumstances in which specifying transition rates is easier than directly specifying macroeconomic behavior for the whole collection of microeconomic agents. In the applications of Chapter 5, the state space is discrete, or agents' choice sets are discrete, and the degree of knowledge of the system is such that the microeconomic states of the system are the vectors of the fractions of agents with the same decisions or microeconomic states, as the case may be. At whatever level of detail we wish to describe stochastic dynamics, the master equation, which is a version of the backward Chapman–Kolmogorov equation, governs time evolution of the probabilities of the states in the form of probability flows into and out of them.

How do we construct macroeconomic models? Microeconomic specifications of agent interactions imply macroeconomic models via the process of aggregations that is implemented in this book. There is some superficial similarity between our specification of transition rates for jump Markov processes and fitness functions in the evolutionary game literature to express the idea that better choices win over converts, but there are some essential differences. For example, we do not impose deterministic rules for strategy mixed as in the game literature, but rather examine stochastic processes for this conversion process by backward Chapman–Kolmogorov equations which govern the time evolution of the probability distribution of the mixes of agents or strategies.

We develop two novel aggregation procedures in Chapters 5 and 7 in order to obtain macroeconomic dynamics of a large collection of interacting microeconomic units. One is via the master equation (Chapman–Kolmogorov equation) to describe probability flows of agents moving among states. Specifying behavior of microeconomic units in terms of transition probabilities or rates is a useful way of constructing macroeconomic models when certain technical conditions, called the detailed (or partial) balance conditions, are met, because the equilibrium probability distributions, called Gibbs distributions, then exist. The first term of an approximate expansion solution method of the equation yields the dynamic equation for the aggregate or macroeconomic variables. This is one way, first introduced in Chapter 4 and further developed in Chapter 5.

The aggregate or macroeconomic equations keep track of the time evolution of the expected fractions or something like conditional means of the population, whereas fluctuations about the means are described in some cases by Fokker–Planck equations.

The second way to derive aggregate models is via renormalization group theory associated with hierarchy, which is discussed in Chapter 7. By adopting a more disaggregate and explicitly stochastic dynamic framework than is traditional in economic modeling, we demonstrate in Chapters 4, 5, and 7 that we have new ways of aggregating microeconomic units, and can explain sluggishness of responses of some macroeconomic phenomena. We discuss interactions among microeconomic agents in Chapters 4 through 7. These two novel aggregation procedures allow us to examine macroeconomic dynamics of a large collection of interacting microeconomic units.

To summarize, we model agents as interacting jump Markov processes, and the dynamic version of market participation models and other discrete-choice models are put in this framework. Our reformulation of the macroeconomic modeling procedure involves recasting the deterministic models into a stochastic framework of jump Markov processes in some of the examples cited above. In other cases where the original formulations are already in stochastic terms, there is a need to recognize or reformulate the problems as those involving jump Markov processes. Specifics of how these reformulations are carried out, of course, vary from problem to problem. All involve specifying interactions of microeconomic units probabilistically in terms of transition probabilities (discrete time) or transition rates (continuous time) of Markov chains, and use a version of Chapman–Kolmogorov equations to derive time evolutions of probability distributions of state variables, suitably defined. See Sections 4.7 and 5.1 for further discussions of the basic ingredients of such reformulations.

This book illustrates that our framework goes some way in improving the state of economic modeling. Concepts and techniques that are not in the toolkit of traditional macroeconomists also are developed in this book to support this framework. New model construction methods for interacting microeconomic units that we advocate in this book give us results that have not been possible in the traditional framework of deterministic and representative agents.

Having decided to use jump Markov processes to model microeconomic agents, let us next mention the types of interaction among agents considered in this book.

In Chapter 4, we introduce jump Markov processes to pave our way for modeling a large collection of interacting agents. We defer to Chapter 6 our discussion of pairwise interaction or interaction with randomly matched (drawn) anonymous microeconomic units. In Chapter 5, we concentrate on the type of interaction or externality that does not lend itself to modeling with pairwise

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interaction. We call these externalities **mean-field effects** or simply **field effects**. This terminology is chosen to convey the notion that interaction with a whole population or class of microeconomic units is involved, that is, aggregate (macroeconomic) effects due to contributions from a whole population of microeconomic units or composition of the whole in the sense that fractions of units in various states or categories are involved.

A good case in which to examine the field effects is a study of the dynamic behavior of a group of agents who interact through the choices they make in circumstances in which each agent has a set of finite decisions from which to choose, and in which they may change their minds as time progresses, possibly at some cost. In such situations, an agent's choice is influenced by the vector of the fractions of agents who have selected the same decisions, because the agent's perceived profit or benefit from a decision change will be a function of this vector, that is, the composition of agents with the same decisions in the whole population of agents.

1.4 Hierarchical state spaces

We have mentioned the multiplicity of microeconomic states and multiple equilibria. The third consequence of large models is the complexity of dynamic behavior in a large dimensional state space, or complex behavior of performance indexes or cost structures of dynamic models in such large dimensional state spaces. The problems are similar to those of complex landscapes of performance indexes in difficult optimization problems or energy landscapes of large neural networks, the kind that requires sophisticated numerical optimization algorithms, such as simulated annealing, in seeking optimal solutions. One way to think of hierarchical or tree-structured state space is to recognize that some subsets of state spaces with high dimensions are rarely occupied by models, and trees are introduced to approximate such state spaces.[6]

Large state spaces without any structure are difficult to deal with. The state spaces of our models, however, have some structures which we exploit in modeling. These state spaces are organized into tree or hierarchical structure, and we introduce the notion of ultrametrics or tree distance to discuss similarity of states in nodes of trees. This is the way we deal with complex landscapes of performance indexes or cost structures of large models. These are introduced in Chapter 2, and discussed in detail in Chapter 7.

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[6] Doyle and Snell (1984) have an interesting discussion of approximating random walk in the three-dimensional space by a tree space. Another way to introduce trees is to map local minima and maxima of functions with complex behavior into a tree as in Aldous and Vazirani (1993).

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1.5 Scope of modeling

The proposed modeling procedure can be applied whenever and wherever specifications of optimization problems or behavior rules for microeconomic agents can be transformed into those of transition rates between configurations of microeconomic states of a collection of such agents. Some indications of how such transformations can be accomplished are given in Chapters 4 through 6. Chapter 5, in particular, shows by examples how the knowledge or specifications of transition rates allows us to specify the dynamics as the master equations for a composition of the population of microeconomic agents. In general, the master equations do not admit closed-form solutions. In Chapter 5 we give some of the approximate-solution methods of van Kampen (1965, 1992) and Kubo (1975). We also show the reader the importance of the multiplicity of microeconomic agents in interpreting the equilibrium probability distributions of the master equations. Whenever the vectors of fractions of agents are appropriate microeconomic states for interacting agents, we can employ the master equations to describe time evolution of the population. The proposed approach, therefore, is limited only to the extent that the equations prove to be analytically or computationally intractable.

One of the major aims of this book is to examine consequences of changing patterns of agent interaction over time. We view these patterns as distributions of agents over possible types or categories over time, and we endevour to discover stable distributions of patterns of interactions among agents.

Equilibrium distributions of patterns of classification describe stable emergent properties of the model. Some simple and suggestive examples are discussed in Chapters 4 and 5.

In models with a small number of types, choices or classes, the methods developed in Chapter 5 are quite effective. When the number of classes becomes large, (numerical) solutions of the master equations may become cumbersome. An attractive alternative is to examine equilibrium distributions directly. We outline this method in the concluding section of this book by drawing on the literatures of population genetics and statistics.

CHAPTER 2

Simple illustrative and motivating examples

This chapter introduces concepts and techniques that are further developed in the main body of this book, mostly via simple examples. The objective is to introduce to the reader our approaches, viewpoints, and techniques of modeling that are possibly unfamiliar to the economics profession, and to illustrate them in simple context so that basic ideas can be easily grasped. For simplification, some examples are artificial, but suggestive enough to demonstrate relevance of the notions or methods in more realistic macroeconomic models.

The examples focus on the statistical, dynamic, and state-space properties of economic variables and models. They help to introduce to the reader the notions of distinguishable and exchangeable microeconomic agents, multiplicity of microeconomic states and entropy as a measure of multiplicity, empirical distributions and related topics such as Sanov's theorem and the Gibbs conditional principle, and dynamics on trees, among others.

All of these examples are intended to suggest why certain concepts or modeling techniques, possibly not in the mainstream of the current macroeconomic literature, are useful in, or provide a new way of examining, aggregate behavior of a large number of interacting microeconomic agents.

2.1 Stochastic descriptions of economic variables

We treat all micro- and macroeconomic variables as random variables or stochastic processes, although macroeconomic variables become deterministic in the limit of the number of microeconomic units approaching infinity. Relationships among economic variables are statistical in an essential way, and are not made so by having additive disturbances or measurement errors superimposed on deterministic relationships.

In this book we consistently model economic systems as stochastic, not as deterministic. As mentioned in Chapter 1, if time evolution of an economy is modeled as the outcome of a deterministic process, then different locally