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Aspects of Galois Theory

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INTRODUCTION

This volume grew out of the 'UF GALOIS THEORY WEEK', a conference held at the University of Florida, Oct. 14–18, 1996. The conference was dedicated to the Inverse Galois Problem. The richness of this area stems from the fact that it attracts people from all kind of mathematical backgrounds, with methods ranging from explicit polynomial calculations and even numerical computer calculation to the structure and character theory of finite simple groups and up to the most abstract methods of algebraic/arithmetic geometry (like moduli stacks). In the original spirit of Galois, all these turn out to be just different aspects of the same matter.

Here is a brief description of the contents of this volume. Abhyankar continues his work on explicit classes of polynomials in characteristic $p > 0$ whose Galois groups comprise entire families of Lie type groups in characteristic p. In characteristic > 0, such polynomials have so far only been found for finitely many non-abelian simple groups except alternating groups. New methods for this, involving the determination of explicit equations for certain Hurwitz spaces, are developed in Couveignes' paper.

There is a way of realizing infinite series of Lie type groups as Galois groups over the rationals, based on Riemann's existence theorem and the notion of rigidity. It does not directly yield polynomials having these groups as Galois groups, however. New results in that direction are obtained in the paper of Thompson and Völklein, using a certain generalization of rigidity called the braid-abelian property and resulting in Galois realizations of the projective symplectic groups $\mathrm{PSp}(n, q)$ under certain restrictions on n and q.

The more abstract aspects come into play when considering the totality of Galois extensions — with certain properties — of a given field. This amounts to the study of certain (profinite) fundamental groups and absolute Galois groups. Harbater studies the fundamental group of an affine curve in positive characteristic. He determined the finite quotients of this fundamental group in his proof of Abhyankar's conjecture, but its full structure as profinite group remains mysterious. Ihara investigates the relationship between the absolute Galois group of the rationals and the so-called Grothendieck-Teichmüller group. The former is an arithmetic object, controlling the totality of Galois extensions of the rational field, whereas the latter is a geometric object (a certain subgroup of the automorphism group of the geometric fundamental group of the 3-punctured line). Ihara's paper provides evidence that these two groups may not be as closely related as was previously hoped for. Nakamura's article surveys related material.

Wewers' paper attempts to provide a bridge between the abstract and concrete approaches, by supplying a more elementary proof of Grothendieck's results on the relationship between the fundamental group of the punctured line in characteristic 0 and in positive characteristic. Dèbes studies general

questions concerning the field of definition of a cover and models of the cover over such a field.

The articles of Frey/Kani/Völklein and of Müller apply methods developed for the Inverse Galois Problem to other areas. The former constructs infinite towers of unramified Galois curve covers defined over a fixed number field or finite field, with the additional property that there is a compatible system of rational points on these curves. In particular, this provides families of curves with 'many' rational points. Finally, Müller's paper investigates arithmetic properties of rational functions related to Hilbert's irreducibility theorem.