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London Mathematical Society Lecture Note Series. 259

Models and Computability

Invited papers from Logic Colloquium '97 - European Meeting of the Association for Symbolic Logic, Leeds, July 1997

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521635509

First published 1999

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-63550-9 Paperback

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Preface

Basic science, and within that pure mathematics, has a unique ability to surprise and change our view of the world we live in. But more often than not, its fundamental 'relevance' has emerged in ways impossible to have anticipated. As has often been remarked, that of the best basic science (say of non-Euclidean geometry, or of Hilbert spaces, or of the universal Turing machine) is independent of limited views of potential applicability.

Logic Colloquium '97, held in Leeds, England, 6th – 13th July, 1997, set out to reflect all that was best in contemporary logic, and **Models and Computability** and **Sets and Proofs** comprise two volumes of refereed articles, mainly based on the invited talks given at that meeting. Thanks to the programme committee (its other members being George Boolos, Sam Buss, Wilfrid Hodges, Martin Hyland, Alistair Lachlan, Alain Louveau, Yiannis Moschovakis, Leszeck Pacholski, Helmut Schwichtenberg, Ted Slaman and Hugh Woodin) and the special sessions organisers (Klaus Ambos-Spies, Sy Friedman, Wilfrid Hodges, Gerhard Jaeger, Steffen Lempp, Anand Pillay and Helmut Schwichtenberg), the editors have been able to call on a rich and distinguished array of authors. It is of great regret that one of our programme committee members was not able to see the success to which he had substantially contributed, and the **British Logic Colloquium Lecture**, given by Paul Benacerraf, took the form of a tribute to his memory. It would be difficult for us to improve on the introduction to Professor Benacerraf's article (on p.27 of **Sets and Proofs**) provided by the following extract from the comments received from the referee (necessarily anonymous):

> 'Ever since Paul Benacerraf published "What numbers could not be" (1965) and "Mathematical truth" (1973), his views have been seminal in the development of philosophy of mathematics, and for this reason one can expect that any paper by him that revisits the issues he first discussed in those papers (see footnote 1) will be of immediate interest for the subject. Also, the present paper is written as a very personal tribute to George Boolos, a student of Paul Benacerraf, whose early death, at the age of 55, deprived philosophy of mathematics of one of its leading – and one may also say – best loved contributors, and the Association of Symbolic Logic of its serving President, so it is very particularly fitting that this paper should be published in the proceedings of a meeting of the ASL.'

Logic Colloquium '97 was also the first such conference in Britain since the death of Robin Gandy (the first president of the British Logic Colloquium) on 20

November, 1995. As observed by Andrew Hodges[1]:

> 'Robin Gandy's death on 20 November 1995 has ended the strongest living link with Alan Turing, with whom he was all of intimate friend, student and colleague. He inherited all Turing's mathematical books and papers; and thereafter also carried forward part of Turing's intellectual tradition; more precisely he took on the subject that Turing lost interest in, by becoming a pre-eminent British figure in the revival and renewal of mathematical logic.'

Appropriately, Gerald Sacks, Robin's long-time friend and occasional research collaborator, gave the **Robin Gandy Lecture** (eloquently introduced by Joe Shoenfield). The original invitation to Professor Sacks had suggested a theme of "Computability Theory – The First Sixty Years", perhaps with Robin's 1988 article on "The Confluence of Ideas in 1936" in mind. But Gerald, undoubtedly grand but never grandiose, responded in his own, very personal, way, and the paper based on his lecture (on pages 367–376 of this volume) provides, among other things, fascinating background to the development of such landmarks of contemporary computability theory as the Sacks Density Theorem.

An element of arbitrariness in the allocation of topics and particular papers between the two volumes has been unavoidable. The prominence of effective model theory as a conference topic was one determining factor in the particular distribution adopted. Other decisions, such as the inclusion of the two papers from the 'Philosophy of Proof' part of the conference programme in **Models and Computability**, were more practical in origin. We hope that the overall benefits of convenience to the reader will be sufficient compensation.

Together, we hope that this volume and its companion **Sets and Proofs** will provide readers with with a comprehensive guide to the current state of mathematical logic, and while not pretending to the definitiveness of a handbook, perhaps communicating more of the excitement of a subject in flight. All the authors are leaders in their fields, some articles pushing forward the technical boundaries of the subject, others providing readable and authoritative overviews of particular important topics. (All the contributors have been encouraged to include a good introduction, putting their work in context.) A number of papers can be expected to become classics, essential to any good library (individual or institutional).

In any project of this magnitude, it is impossible to thank all those who have helped. Special thanks are due to all the authors (and to the small number who tried, and failed to deliver!), and to the host of referees who coped with tight deadlines without complaint. On the technical side, we would like to thank Margaret Williams, Audrey Landford, Tim Hainsworth, Frank Drake, David Knapp, Zarina Akhtar, Eric Cole, Kevin McEvoy, and Ben Salzberg and Benjamin Thoma at Blue

[1] At http://www.turing.org.uk/turing/scrapbook/robin.html.

Sky Research. Finally, thanks to Rebecca Mikulin and Roger Astley at Cambridge University Press for their advice and inexhaustible (it seemed) patience.

All royalties accruing from the sale of **Models and Computability** and **Sets and Proofs** go directly to the British Logic Colloquium.

We dedicate this volume to the memory of Robin Gandy.

S. Barry Cooper
John K. Truss
Leeds, November 1998