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Index of symbols. These are grouped according to the type of object to which they most closely relate.

Categories

- $[n]$: $\{0, \dots, n\}$ as a category, 37
 $[\mathcal{C}, \mathcal{D}]$: functor category, 36
 $\bigoplus_{\lambda} \mathcal{C}_{\lambda}$: direct sum of categories, 108
 $\prod_{\lambda} \mathcal{C}_{\lambda}$: product category, 13
 $\mathcal{C} \setminus \mathcal{F}$: right-fibre category, 24
 $\mathcal{C} \setminus \mathcal{C}$: slice category, 24
 $\text{Sk}(\mathcal{C})$: skeleton of \mathcal{C} , 39
 $\mathcal{A}\mathcal{B}$: category of abelian groups, 3
 \mathcal{A}/\mathcal{C} : quotient category, 281
 $\text{ADD}(X)$: additive category generated by X , 92
 $\mathcal{A}\mathcal{R}\mathcal{T}\mathcal{R}$: category of Artinian modules, 102
 $\mathcal{A}\mathcal{S}\mathcal{S}\mathcal{R}$: Artinian semisimple modules, 90
 $\mathcal{A}\mathcal{U}\mathcal{T}\mathcal{C}$: automorphism category, 14
 $\mathcal{B}\mathcal{A}\mathcal{L}\mathcal{R}$: category of balanced bimodules, 160
 $\mathcal{B}\mathcal{A}\mathcal{S}\mathcal{E}\mathcal{S}\mathcal{R}$: based free modules of finite rank, 32
 $\mathcal{B}\mathcal{A}\mathcal{S}\mathcal{E}\mathcal{S}\mathcal{R}^{\infty}$: based free modules, 44
 $\mathcal{B}\mathcal{G}$: group as a category, 14
 ${}_{\mathcal{R}}\mathcal{B}\mathcal{I}\mathcal{M}\mathcal{O}\mathcal{D}\mathcal{R}$: bimodule category, 160
 $\mathcal{B}\mathcal{R}\mathcal{A}\mathcal{N}\mathcal{D}\mathcal{T}(\Sigma)$: Brandt groupoid, 219
 $\mathcal{C} \times \mathcal{D}$: product category, 11
 $\mathcal{C}, \mathcal{D}, \dots$: categories, 2
 \mathcal{C}^{Λ} : product of copies of \mathcal{C} , 13
 \mathcal{C}^{\odot} : mirror category, 6
 \mathcal{C}^{\oplus} : split exact category, 120
 \mathcal{C}^{op} : opposite category, 8
 \mathcal{C}^n : product of copies of \mathcal{C} , 13
 $\mathcal{C}_1 \times \dots \times \mathcal{C}_n$: product, 13
 $\mathcal{C}\mathcal{A}\mathcal{T}$: category of categories, 23
 $\mathcal{C}\mathcal{O}\mathcal{K}(\gamma)$: in which $\text{Cok}(\gamma)$ is initial, 60
 $\mathcal{C}\mathcal{O}\mathcal{P}\mathcal{R}\mathcal{D}(\mathcal{C}(\Lambda))$: in which the coproduct is initial, 58
 $\mathcal{C}\mathcal{O}\mathcal{P}\mathcal{R}\mathcal{D}(\mathcal{C}', \mathcal{C}'')$: in which the coproduct is initial, 58
 $\mathcal{E}\mathcal{G}$: group as a category, 14
 $\mathcal{E}\mathcal{N}\mathcal{D}\mathcal{C}$: endomorphism category, 14
 $\mathcal{E}\mathcal{N}\mathcal{S}$: category of sets, 3
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 ${}_{\mathcal{R}}\mathcal{F}$: free left modules, finite rank, 10
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 $\mathcal{F}_n \mathcal{C}$: category of sequences, 132
 $\mathcal{F}\mathcal{A}\mathcal{B}$: finite abelian groups, 90
 $\mathcal{F}\mathcal{I}\mathcal{E}\mathcal{L}\mathcal{D}$: category of fields, 48
 $\mathcal{F}\mathcal{R}\mathcal{E}\mathcal{E}\mathcal{R}$: free right modules, 10
 ${}_{\mathcal{R}}\mathcal{F}\mathcal{R}\mathcal{E}\mathcal{E}$: free left modules, 10
 ${}^* \mathcal{F}\mathcal{R}\mathcal{E}\mathcal{E}\mathcal{R}$: based free modules, 44
 $\mathcal{G}\mathcal{P}$: category of groups, 3
 $\mathcal{H}\mathcal{A}\mathcal{B}$: Hausdorff topological abelian groups, 88
 $\mathcal{I}\mathcal{N}\mathcal{F}\mathcal{A}\mathcal{B}$: infinite abelian groups, 60
 $\mathcal{I}\mathcal{N}\mathcal{J}\mathcal{R}$: category of injective modules, 80
 $\mathcal{I}\mathcal{S}\mathcal{O}\mathcal{C}$: isomorphism category, 14
 $\mathcal{K}\mathcal{E}\mathcal{R}(\gamma)$: in which $\text{Ker}(\gamma)$ is terminal, 59
 $\mathcal{M}_{\mathcal{R}, \mathcal{O}}$: relative category, 161
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 ${}_{\mathcal{R}}\mathcal{M}$: finitely generated left modules, 10
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 ${}_{\mathcal{R}}\mathcal{M}\mathcal{O}\mathcal{D}$: category of left R -modules, 3
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 $\mathcal{M}\mathcal{O}\mathcal{R}(W; R, S)$: relative morphism category, 148
 $\mathcal{M}\mathcal{O}\mathcal{R}\mathcal{C}$: morphism category, 13
 $\mathcal{M}\mathcal{O}\mathcal{R}^2\mathcal{C}$: morphisms of morphisms, 16
 $\mathcal{N}\mathcal{I}\mathcal{L}\mathcal{R}, \mathfrak{a}$: nilmodules, 286
 $\mathcal{N}\mathcal{O}\mathcal{E}\mathcal{T}\mathcal{H}\mathcal{R}$: Noetherian modules, 102
 $\mathcal{O}\mathcal{R}\mathcal{D}$: category of ordered sets, 4

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$\mathcal{PRD}(C(\Lambda))$: in which the product is terminal, 58
 $\mathcal{PRD}(C', C'')$: in which the product is terminal, 57
 $\mathcal{PREADD}(X)$: category generated by X , 85
 \mathcal{PROJ}_R : projective right modules, 10
 \mathcal{RPROJ} : projective left modules, 10
 \mathcal{RING} : category of rings, 3
 \mathcal{RNG} : category of nonunital rings, 3
 \mathcal{SET} : category of sets, 3
 $\mathcal{STANBASES}_R^\infty$: free modules with standard bases, 44
 $T_{A(\mathcal{O}), \mathcal{O}}$: torsion adèle modules, 306
 $T_{A(\mathcal{O}), \mathfrak{p}}$: primary adèle modules, 306
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 \mathcal{TOP} : category of topological spaces, 4
 \mathcal{TOPAB} : topological abelian groups, 88
 $\mathcal{TOR}_{\mathcal{O}, R}$: relative torsion modules, 130
 $\mathcal{TOR}_{\mathcal{O}}, \mathcal{TOR}_{\mathfrak{p}, R}, \mathcal{TOR}_{\mathcal{O}, R}$: categories of torsion modules, 267

Exact sequences

$C(M, \mathbf{E})$: measures non-exactness of Hom sequence, 335
 $\text{Ex}(C)$: specified exact sequences, 119
 $\text{Ext}_R^1(M'', M')$: group of extensions, 81
 $\phi_* \mathbf{E}$: push-out of sequence, 80
 $\theta^* \mathbf{E}$: pull-back of sequence, 80

Functors

η : natural transformation, 26
 id_F : identity natural transformation, 36
 \cong :
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 $-\otimes_R -$: tensor bifunctor, 142
 $-\otimes_R N$: tensor functor, 140
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 $\text{Cnst}_D, \text{Cnst}_{\mathcal{D}}$: constant functors, 43
 $\text{Ext}_R^n(-, -)$: extension bifunctors, 77
 $f^\#$: restriction of scalars, 165
 $f_\#$: extension of scalars, 166
 $F(C_1, \dots, C_{h-1}, -, C_{h+1}, \dots, C_k)$:
 restriction of multifunctor, 35
 Fr : free object functor, 19

$GF, G \circ F$: composite or product of functors, 19, 20
 $H^M(-), H_X(-)$: Hom functors, 76, 77
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 Inc : inclusion functor, 18
 lim^1 : derived functor, 250
 $\overleftarrow{\text{lim}}$: derived functor, 250
 Mir : mirror functor, 21
 $M \otimes_R -$: tensor functor, 141
 $\text{Mor}_C(-, -)$: morphism bifunctor, 33
 $\text{Mor}(-, X), \text{Mor}(L, -)$: morphism functors, 22
 Op : opposite functor, 18
 $P: R \mapsto \langle 1_R \rangle$: ring to prime subring, 28
 $\text{Tor}_n(-, -)$: Tor bifunctors, 153
 Υ : forgetful or underlying functor, 18

Groups

$ghg^{-1}h^{-1}$: commutator, 25
 $\langle T \rangle$: monoid generated by T , 254
 $[C]$: element in Grothendieck group, 123
 $\text{Cl}(\mathcal{O})$: ideal class group, 217
 $\text{Fr}(C)$: free abelian group on isomorphism classes, 123
 G_{ab} : commutator quotient group, 25
 $[G, G]$: commutator subgroup, 25
 $G' * G''$: group free product, 58
 $\text{GL}(R)$: stable general linear group, 229
 $\text{GL}_n(R)$: general linear group of degree n , 29
 H^G : normal closure, 64
 $K_0(C), K_0(C, \text{Ex}(C))$: Grothendieck group, 123
 $\text{Pic}(R)$: Picard group, 214
 $\text{Rel}(C)$: relation subgroup, 123
 S_n : symmetric group, 16

Homomorphisms and morphisms

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 ι_C : morphism from initial object, 48
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 $\iota^{\wedge \mu}$: \leq as a morphism, 4
 λ^*, ξ_* : induced maps on morphisms, 23
 $\text{Mor}_C(C, D)$: morphisms in a category, 2

$\text{Mor}(L, \xi), \text{Mor}(\lambda, X)$: induced maps on morphisms, 22

Ideals

$\{a\}$: class of a in $\text{Cl}(\mathcal{O})$, 109
 ab : product, 109
 $\text{Ann}(M)$: annihilator, 111
 $\text{Cl}(\mathcal{O})$: ideal class group, 109
 $c(S, R)$: conductor, 339
 $\text{Frac}(\mathcal{O})$: fractional ideals, 109
 $\text{In}(R)$: invertible fractional ideals, 220
 $n(L, M)$: an invariant, 321
 $\text{Pr}(\mathcal{O})$: principal ideals, 109
 $\text{rad}(R)$: Jacobson radical, 103

Matrices

$A \oplus 1$: augmented matrix, 29
 A^t : transposed matrix, 29
 $M_{m,n}(R)$: $m \times n$ matrices, 145
 $M_n(R)$: matrix ring, 28

Measuring functions

$\|\cdot\|$: norm, 310
 $d(r, s)$: metric, 310
 $\text{pd}_R(L)$: projective dimension, 342
 v_p : p -adic valuation, 292

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 $M \otimes_R N$: tensor product, 136
 $M \times_{M''} L''$: pull-back, 67
 $M' \oplus M''$: direct sum, 91
 $M' \oplus_L L$: push-out, 65
 M^G : induced module, 178
 $M_{\mathfrak{p}}$: localization at \mathfrak{p} , 264
 $M\mathfrak{a}$: module times ideal, 169
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 M^n : direct sum of copies of M , 144
 MS : $M \otimes_R S$ when M_R is flat, 169
 MR_Σ : M_Σ for R torsion-free, 269
 M_Σ : localization of module, 264
 N_H : restricted module, 178
 $\text{rad}(L)$: radical of a module, 208
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SN : $S \otimes_R N$ when ${}_R N$ is flat, 169
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 $T_{\mathfrak{p}}(M)$: \mathfrak{p} -primary component, 111
 $W^* = \text{Hom}({}_R W, {}_R R)$: dual, 185
 $W^{**} = \text{Hom}(W_R^*, {}_R R)$: double dual, 186

Objects in categories

$C' \amalg C''$: coproduct, 58
 $\coprod_{\Lambda} C_\lambda$: coproduct of objects, 59
 $C' \prod C''$: product, 57
 $\prod_{\Lambda} C_\lambda$: product, 58
 \prod' : restricted direct product, 305
 (C, D, χ) : morphism from C to D , as object in morphism category, 13
 $[c_\lambda]$: image in direct limit, 226
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 $(\text{Cok } \gamma, \chi)$: cokernel, 60
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 $D(M, N)$: denominator set, 279
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 $\overline{\mathbb{F}}_p$: algebraic closure of \mathbb{F}_p , 228
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 $\mathcal{D}(T, \alpha)$: skew rational functions, 271
 $\mathcal{F}[[T]]$: formal power series, 302
 $\widehat{\mathcal{K}}$: complete field, 297
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 $\mathcal{O}_{\mathfrak{p}}$: localization, 254
 $\mathcal{O} \setminus \mathfrak{p}$: difference set, 254
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 \mathcal{V} : valuation ring, 291
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 $A(\mathcal{O})$: adèle ring, 305
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 CR : cone of a ring, 288
 mR : nonunital ring of all finite matrices, 229
 $M_n(R)$: matrix ring, 28
 \bar{R} : unitalization, 28
 R_p : localization of an order, 254
 R_Σ : ring of fractions, 256
 ΣR : ring of left fractions, 272
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 $\text{Latid}(R)$: ideal lattice, 237
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