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Index of symbols. These are grouped according to the type of object to which they most closely relate.

Groups

\dagger : operation on quasi-invertible elements, 13

$\prod_{i \in I} G_i$: product of groups, 45

$g^0 = 1, g^{-i} = (g^{-1})^i$, 194

$GL_n(R)$: general linear group, 9

$QU(R)$: quasi-invertible elements, 13

$U(R)$: units of a ring, 9

Homomorphisms

\cong : isomorphism, 6

$\alpha_1 \oplus \cdots \oplus \alpha_k$: direct sum of homomorphisms, 42

χ : characteristic homomorphism, 6

$F(\lambda)$: induced map on torsion-free quotient, 247

id_M : identity homomorphism, 19

ι : a canonical embedding, 47

$\iota: R \rightarrow M_s(R)$, diagonal map, 62

$T(\lambda)$: induced map on torsion part, 247

$T_p(\lambda)$: induced map on primary component, 245

$\text{Hom}(M, N), \text{Hom}({}_R M, {}_R N)$: homomorphisms of (left) modules, 19

$\text{Hom}(M_R, N_R)$: homomorphisms of right modules, 18

Ideals

$r \equiv s \pmod{\mathfrak{a}}$: congruence, 7

$\{\mathfrak{a}\}$: class of \mathfrak{a} , 191

(x_1, \dots, x_n) : ideal, 5

$1 + j, j$ an ideal, 176

$\mathfrak{a}, \mathfrak{b}, \dots$: ideals, 4

$\mathfrak{a} \mid \mathfrak{b}$: divides, 193

$\mathfrak{a}^2, \mathfrak{a}^k$: powers of ideal, 179

ab : product ideal, 5, 189

$c(\mathfrak{p})$: complement of \mathfrak{p} , 243

$\text{Ann}(x)$: annihilator, 24

$\text{Fit}_k(A)$: Fitting ideal, 141

$\text{Frac}(\mathcal{O})$: fractional ideals, 189

$\text{In}(R)$: invertible ideals, 200

$\text{Ker } f$: kernel of ring homomorphism, 6

mR : infinite matrices, almost all entries 0, 171

\mathbf{P} : nonzero prime ideals, 194

\mathbf{Pe} : representative nonzero primes (irreducibles), 196

$\text{Pr}(\mathcal{O})$: principal ideals, 189

Matrices

$(m)^F$: coordinate vector, 65

$F(m)$: coordinate vector, 56

A^t : transpose of a matrix, 34

$A^{FG}(\alpha)$: matrix of a homomorphism, 66

$A_{GF}(\alpha)$: matrix of a homomorphism, 58

$M_{t,s}(R)$: set of $t \times s$ matrices, 58

$\mu_F(\alpha)$: matrix with respect to F , 60

$\mu^F(\alpha)$: matrix with respect to F , 66

Modules

$\bigoplus_I L_i$: external direct sum, 44

$\bigoplus_\Lambda L_\lambda$: external direct sum, 46

$M = \bigoplus_{i \in I} M_i$: (internal) direct sum, 43

$M = \bigoplus_{\lambda \in \Lambda} M_\lambda$: (internal) direct sum, 46

$\prod_I L_i$: direct product, 44

$\sum_{i \in I} M_i$: sum of submodules, 21

$\alpha M'$: image of submodule, 23

$\alpha^{-1} N'$: inverse image of a submodule, 23

α_i : multiplicity of a component, 241

$\text{Cok } \alpha$: cokernel, 76

$\text{edt}(M)$: elementary divisor type, 245
 $\text{edt}_p(M)$: elementary divisor type, 241
 $\text{Fr}_R(X)$: standard free module, 47
 $\text{Im } \alpha$: image of module homomorphism, 22
 $\mathcal{I}(R)$: representative irreducible modules, 151
 $\text{Ker } \alpha$: kernel of module homomorphism, 22
 $L^{(\Lambda)}$: direct product, 46
 L^Λ : direct sum, 46
 L^k : direct sum, 53
 $L_1 \odot \cdots \odot L_k$: external direct sum, 40
 $m + M = \bar{m}$: equivalence class, 22
 ${}_R M$: left module, 16
 ${}_R M_S$: bimodule, 17
 M/M' : quotient module, 22
 M° : opposite module, 19
 $\{M\}$: ideal class of a module, 228
 $M_1 + \cdots + M_k$: sum of submodules, 21
 $M_1 \oplus \cdots \oplus M_k$: direct sum, 37
 M_R : right module, 16
 $M\alpha$: module times ideal, 24
 $\text{mult}(M)$: multiplicity type, 151
 R^Λ : free module, 47
 R^n : free right module, 17
 ${}^n R$: free left module, 17
 $(\Lambda)R$: direct product, left module, 47
 ${}^\Lambda R$: direct sum, left module, 47
 $\text{rad}(M)$: radical, 173
 $\text{SP}_R(X) = \sum_{i \in I} x_i R$: span, 21
 $T(M)$: torsion submodule, 238
 $T_p(M)$: p -component, 245
 $T_p(M)$: p -primary component, 243

Number theory, factorization

$\|$: total division, 126
 $\|a\|, \|a\|$: norm, 215
 $[\mathcal{K} : \mathcal{Q}]$: degree, 201
 $C_x(X)$: characteristic polynomial, 204
 $\text{Cl}(\mathcal{O})$: ideal class group, 191
 (d/p) : Legendre symbol, 214
 γx : conjugate of x , 204
 Nx : norm, 204
 Tx : trace, 204
 \hat{p}, p_1, p_2 : primes above p , 213
 $v(p, x)$: exponent of p in x , 197
 $v(\mathfrak{a})$: valuation, 231
 $v(p, \mathfrak{a})$: exponent in \mathfrak{a} , 194
 (I) : inert, 213
 (R) : ramified, 212
 (S) : split, 212

Rings

\mathbb{C} : the complex numbers, 2
 \mathbb{H} : quaternions or Hamiltonians, 67
 \mathbb{N} : the natural numbers, 26

\mathbb{Q} : the rational numbers, 2
 \mathbb{R} : the real numbers, 2
 \mathbb{Z} : the integers, 2
 $0_R, 1_R$: zero, identity of ring R , 3
 $A[\epsilon]$: ring of dual numbers, 79
 AG : group ring, 106
 $A[T, \alpha]$: skew polynomial ring, 116
 $A\langle X_1, \dots, X_k \rangle$: noncommutative polynomials, 11
 $C(a, \alpha)$: skew centralizer, 172
 CR : cone of R , 70
 \mathcal{D}^α : invariant elements, 124
 $\text{End}(M_R)$: endomorphism ring of M_R , 18
 $\text{End}({}_R M)$: endomorphism ring of ${}_R M$, 19
 H : Hurwitz quaternions, 130
 $\text{Im } f$: image of ring homomorphism, 6
 \mathcal{K} : a field, 9
 $\mathcal{K}(T, \alpha)$: skew rational functions, 118
 $M_\omega^{cc}(E)$: column convergent matrices, 72
 $M_\Lambda^{cf}(R)$: column-finite square matrices, 65
 $M_{\Sigma, \Lambda}^{cf}(R)$: column-finite matrices, 65
 $M_{\Lambda, \Sigma}^{rf}(R)$: row-finite matrices, 66
 $M_{\Lambda}^{rf}(R)$: row-finite square matrices, 66
 $M_n(R)$: matrix ring, 3
 \mathcal{O} :
 a commutative ring, usually a domain, 9
 ring of integers, 203
 \mathcal{O}_p : valuation ring, 232
 $Q(X)$: rational functions, 118
 $\underline{R}, S, T, \dots$: rings, 2
 \bar{R} : unitalization, 13
 R° : opposite ring, 19
 R/α : residue ring, 7
 $R[T]$: polynomial ring, 3
 $T(X)$: tiled matrices, 230
 $T_2(R)$: triangular matrix ring, 86
 $Z(R)$: centre, 4
 \mathcal{Z} : principal ideal domain, 196
 $Z[x]$: x adjoined to \mathcal{Z} , 202

Sequences

\mathbf{E} : short exact sequence, 76
 $\text{Ext}_R^1(M'', M')$: group of extensions, 80
 $\phi_* \mathbf{F}$: push-out of sequence, 83
 $\text{SES}(M'', M')$: set of extensions, 88
 $\theta^* \mathbf{E}$: pull-back of sequence, 83

Sets

C : strict inclusion, 4
 $I \setminus \{i\}$: I without i , 43
 ω : ordered natural numbers, 26

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