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To the memory
of
Hamilton Berrick
(6 March 1987 – 8 September 1994)
who in his short life
brought others so much joy;
with the thought that this book
might one day have given him some pleasure.

Contents

PREFACE

This text is an introduction to the theory of rings and modules for a new graduate student. What gives this book its special flavour is that it is particularly aimed at someone who intends to move on to study algebraic K-theory. This aim influences both the choice of rings and modules that we discuss, and the manner in which we discuss them.

Starting from a knowledge of undergraduate linear algebra together with some elementary properties of the integers, polynomials and matrices, we provide the basic definitions and methods of construction for rings and modules, and then we develop the structure theory for modules over various kinds of ring.

These classes of ring reflect the historical roots of algebraic K-theory in geometry and topology on the one hand, and representation theory and number theory on the other. Thus the rings that interest us are Noetherian rings, in particular skew polynomial rings, Artinian rings, and Dedekind domains.

The text pursues ring and module theory up to the point where the aspiring K-theorist needs category theory. The necessary category theory is dealt with in the companion volume [BK: CM], both in the abstract and in relation to more advanced topics in the ring and module theory. Our division of the subject matter in this way means that the present volume can be regarded simply as an introduction to some fundamental topics in the theory of rings and modules.

Here is a more detailed survey of the material in this text.

The first chapter gives the definitions of rings and modules, together with some examples and constructions. The second chapter develops the groundwork of the theory of modules, with a particular emphasis on the basic ways of constructing and comparing modules. We see how to combine modules to make new ones, using direct sums and, more generally, short exact sequences. The use of short exact sequences to describe the relationships between modules lies at the core of algebraic K-theory, and so the notion of a short exact sequence plays a central role both in this text and in [BK: CM]. Also crucial to K-theory are the free modules, which are assembled by taking direct sums of copies of a ring, and the projective modules, which arise when free modules are in turn split up into direct sums. A projective module over a ring is the appropriate generalization of a vector space over a field.

Other topics that we look at in some depth in the second chapter are the representation of homomorphisms between free modules in terms of matrices, and the use of matrices to relate the different bases of a given free module. This leads to a discussion of the circumstances under which a free module has bases with differing numbers of elements, that is, when the coefficient ring fails to have invariant basis number.

The primary aim of the remainder of this book is to acquaint the reader with some important classes of rings and their module theory, with an emphasis on the description of the projective modules. The prominence of these rings in this text is in part due to their importance in applications, and in part due to the fact that their modules are amenable to calculation. Thus, in Chapter 3 we look at skew polynomial rings and, more generally, noncommutative Euclidean domains. The module theory of such rings is fairly transparent, and the construction of a skew polynomial ring can be iterated to provide some interesting examples of K-groups. Next, in Chapter 4, we investigate the structure of Artinian semisimple rings. These can be characterized by the property that any module is projective. We classify the finitely generated projective modules over such rings. We also look at the structure of Artinian rings in general.

The motivation for our final two chapters is that some of the most interesting results and questions in K-theory originate in number theory. In Chapter 5 we show that a ring of algebraic integers is a Dedekind domain, and we introduce the ideal class group. The theory is illustrated with some explicit computations in quadratic fields. Finally, we show how the projective modules over a Dedekind domain depend on the ideal class group, and we classify all the finitely generated modules over a Dedekind domain.

Since this book is an introduction, we have kept a fairly leisurely pace throughout. There are nearly two hundred exercises, some of which give brief introductions to topics that are not covered in the body of the text.

Our approach takes full advantage of the powerful abstract methods that were introduced in the early part of the 20th century by Emmy Noether and her contemporaries. Thus our treatment of the subject matter does not reflect its historical development. To counterbalance this, we have thrown in the occasional comment on the origins of definitions and results. Many of our comments are based on information obtained from [Bourbaki 1991], [Dedekind 1996], [Srinivasan & Sally 1983] and [van der Waerden 1980], which the reader should consult for fuller details.

This text is divided into chapters and sections. Within each section, all subsections, theorems, propositions, and lemmas are numbered consecutively. Exercises are to be found at the end of each section and have their own separate numbering.

The symbol □ indicates either the end of a proof <u>or</u> that none is needed. Once or twice we use the symbol ◯ after the statement of a result to indicate that the result is not a consequence of the arguments of this text; in this event, a reference is given.

We thank our departments, and Cambridge University Press, for their patience and encouragement during the lengthy gestation period of this text and its companion [BK: CM]. Particular thanks are due to Oliver Pretzel at Imperial whose advice, provided over a long timespan, enabled us to produce this text in LaTeX. We also thank several referees for their comments on drafts of this work; we have made some modifications to take account of their views.

We also thank SERC (Visiting Fellowship GR/D/79586), the London Mathematical Centre, the British Council in Singapore, NUS (Research Grant RP950645), and the Lee Kong Chian Centre for Mathematical Research for providing travel and subsistence expenses so that the authors were able to meet occasionally.

Especially, we thank our families for their tolerance in allowing us to vanish from sight for sporadic and inconvenient periods of very variable lengths, ranging from the odd month to the odd 'five minutes' when something just had to be finished before supper.