Fundamentals of Stellar Evolution Theory: Understanding the HRD

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We summarize the results of stellar evolution theory for single stars that have been obtained over the last two decades, and compare these results with the observations. We discuss in particular the effect of mass loss by stellar winds during the various evolutionary stages of stars that are affected by this phenomenon. In addition, we focus on the problem of mixing in stellar interiors calling attention on weak aspects of current formulations and presenting plausible alternatives. Finally, we survey some applications of stellar models to several areas of modern astrophysics.

1. Introduction

The major goal of stellar evolution theory is the interpretation and reproduction of the Hertzsprung-Russell Diagram (HRD) of stars in different astrophysical environments: solar vicinity, star clusters of the Milky Way and nearby galaxies, fields in external galaxies. The HRD of star clusters, in virtue of the small spread in age and composition of the component stars, is the classical template to which stellar models are compared. If the sample of stars is properly chosen from the point of view of completeness, even the shortest lived evolutionary phases can be tested. The HRD of field stars, those in external galaxies in particular, contains much information on the past star formation history. In these lectures, no attempt has been made to cover all the topics that could be addressed by a report on the progress made in understanding the HRD. Rather we have selected a few topics on which in our opinion most effort has concentrated over the past few years. Among others, the subject of the extension of convective regions in real stars was vividly debated with contrasting appraisals of the problem. Accordingly, various scenarios for the evolution of stars were presented and their far-reaching consequences investigated. Similarly, much effort was dedicated to understanding the evolution of globular cluster stars, and to quantifying the effect of important parameters (see below) in the aim of clarifying whether an age spread is possible.

The lectures are organized as follows; 2. a summary of basic stellar evolution theory, whenever possible updated to include the most recent results; 3. a summary of the physical causes determining violent ignition of a nuclear fuel, core collapse, and explosions. 4. a summary of recent results on relevant nuclear reaction rates and opacities. 5. a critical discussion of stellar winds and their implications on stellar models. 6. a review of the evolution of massive stars under the effect of mass loss. 7. a discussion of several problems related to convective instability and mixing in stellar interiors (semiconvection, overshoot, and diffusion) together with a summary of the evolutionary results under different mixing schemes. 8. a summary of the problems concerning the transformation of the theoretical HRD into the observational color-magnitude diagram. 9. a description of the results obtained for the luminosity functions, age, age-metallicity relation, age spread, and *second parameter* of globular clusters; 10. a discussion of the old open clusters as a means for calibrating the extension of convective cores in the range of low-mass stars; 11. similarly for the rich young clusters of the Large Magellanic Cloud (LMC) but in the range of intermediate-mass stars. 12. a modern description of the properties of supergiant stars in the Milky Way and LMC together with current understanding of their evolution in light of the problems raised by SN1987A. 13. a summary of the most recent developments in modeling the evolution of AGB stars, their luminosity function, and the formation of Carbon stars; 14. a summary of the recent progress made in modeling the pulsation of the Cepheid stars and the specific topics of the shape of the instability strip and mass discrepancy.

For more information the reader is referred to the many review articles that have appeared over the years, including Iben & Renzini (1983, 1984), Iben (1985), Castellani (1986), Chiosi & Maeder (1986), Hesser (1988), Renzini & Fusi-Pecci (1988), Iben (1991), VandenBerg (1991), Demarque *et al.* (1991), Fusi-Pecci & Cacciari (1991), Chiosi *et al.* (1992), Maeder & Conti (1994), Stetson *et al.* (1996), VandenBerg *et al.* (1996), and others quoted in the text.

2. Basic stellar evolution

Independently of the chemical composition, stars can be loosely classified into three categories according to their initial mass, evolutionary history, and final fate: low-mass stars, intermediate-mass stars, and massive stars. Various physical causes concur to define the three groups and related mass limits:

1. The existence of a natural sequence of nuclear burnings from hydrogen to silicon.
2. The amount of energy liberated per gram by gravitational contraction which is increasing with stellar mass.
3. The tendency of the gas in the central regions to become electron degenerate at increasing density.
4. The existence of threshold values of temperature and density in the center for each nuclear step.
5. The relation between these threshold values and the minimum stellar or, more precisely, core mass at which each nuclear burning can start, and the fact that the minimum core mass for a given nuclear burning is not the same for electron degenerate and non-degenerate gas.
6. Finally, the explosive nature of a nuclear burning in a degenerate gas.

Because the evolutionary path of a star in the HRD is a natural consequence of the interplay between those physical processes, they will be the main guide-lines of our summary of the stellar evolution theory.

2.1. *Low-, Intermediate-, and High-Mass Stars: definition*

By low mass stars we define those which shortly after leaving the main sequence toward the red giant branch (RGB), develop an electron degenerate core composed of helium. When the mass (M_{He}) of the He core has grown to a critical value ($0.45 \div 0.50 M_{\odot}$), the precise value depends on the composition, star mass, and input physics), a He-burning runaway is initiated in the core (He-flash), which continues until electron degeneracy is removed. The maximum initial mass of the star (otherwise called M_{HeF}) for this to occur is about $1.8 \div 2.2 M_{\odot}$, depending on the initial chemical composition. Within the same mass range we distinguish the stars lighter than $M_{con} \simeq 1.2 \div 1.3 M_{\odot}$ that burn hydrogen in a radiative core from the more massive ones doing it in a convective core. Furthermore, it is worth recalling that stars lighter than about 0.5 $M_{\odot}$ cannot proceed to central He-ignition because they fail to reach the threshold value for the He-core burning. Stars more massive than M_{HeF} are classified either as intermediate-mass or massive stars. In turn we distinguish the intermediate-mass stars from the massive ones by looking at the stage of carbon ignition in the core. By intermediate mass we mean those stars which, following core He-exhaustion, develop a highly degenerate carbon-oxygen (C-O) core, and as asymptotic giant branch (AGB) stars experience helium shell flashes or thermal pulses. The AGB phase is terminated either by envelope ejection and formation of a white dwarf ($M_{HeF} \leq M_i \leq M_w$) or carbon ignition and deflagration in a highly degenerate core once it has grown to the Chandrasekhar limit of 1.4 $M_\odot$. The limit mass M_w is regulated by the efficiency of mass loss by stellar wind during the RGB and AGB phases (see Iben & Renzini 1983). This point will be discussed in more detail below. The minimum mass of the C-O core, below which carbon ignition in non degenerate condition fails and the above scheme holds, is 1.06 $M_\odot$ corresponding to an initial mass from 7 to 9 $M_\odot$, depending on the chemical composition. This particular value of the initial mass is known as M_{up}. Finally, massive stars are those that ignite carbon nonviolently and through a series of nuclear burnings proceed either to the construction of an iron core and subsequent photodissociation instability with core collapse and supernova explosion ($M_i \geq M_{mas}$), or following a more complicated scheme undergo core collapse and supernova explosion ($M_{up} \leq M_i \leq M_{mas}$). M_{mas} is about 12 $M_\odot$.

Figure 1 shows the evolutionary path in the HRD of model stars of 0.8 $M_\odot$, 5 $M_\odot$, 20 $M_\odot$, and 100 $M_\odot$ which can be considered to be representative of the three categories. These evolutionary tracks have the chemical composition [Z=0.008, Y=0.250] The thick portions of each track approximately indicate the regions of slow evolution, where the majority of stars are observed. The various evolutionary phases discussed in the text are shown as appropriate for the star mass. The reader should refer to this figure to locate on the HRD the particular evolutionary phase under discussion.

2.2. *Core and Shell H-Burning Phases*

The core H-burning main sequence phase of stars lighter than M_{con} is characterized by the gradual formation of a small He core at the center and the buildup of a smooth chemical profile from a He-rich core to the outer unprocessed layers. The luminosity steadily increases while the star climbs along the zero-age main sequence itself departing significantly from it toward cooler T_{eff} only at the very end of the phase. The duration of the core H-burning phase strongly decreases with increasing mass of the star going from about 15×10^9 yr for a typical 0.7 $M_\odot$ star to about 1×10^9 yr for a typical 1.7 $M_\odot$ star.

After the main sequence phase, the H-exhausted core temporarily cools as electron degeneracy sets in, and the energy liberated by gravitational contraction flows out by electron conduction, delaying the increase in central temperature required to ignite helium in the core. As a low-mass star reaches the base of the RGB, the central temperature reaches a minimum approximately equal to the temperature of the H-burning shell. Thereafter, the mass of the helium core grows under the action of the H-burning shell, the core contracts, and temperatures in the core and H-burning shell increase. The luminosity of the star is proportional to the increase in the shell temperature. The rate at which matter is added to the core by the H-burning shell, and consequently the rate of release of gravitational energy and heating of the core, are proportional to the luminosity. The star climbs the RGB (Hayashi line), while convection in the outer layers gets deeper and deeper, eventually reaching those layers that were nuclearly processed in previous stages and generating a discontinuity in the chemical profile (first dredge-up). The steady outward migration of the H-burning shell forces the external convection to recede. The ascent of the RGB is temporarily slowed when the H-burning shell reaches the discontinuity in the chemical profile. Owing to the electron degeneracy, all low-mass stars, independently of initial mass, build up an helium core of approximately the same mass. When this core has grown to about $0.45 \div 0.50 M_\odot$, violent He-burning starts off-center because neutrino emission has cooled the innermost regions (Thomas 1967, Mengel & Sweigart 1981). As the nuclear burning progresses inwards, degeneracy is removed, so that a quiescent nuclear burning in the core begins. The RGB phase is terminated. Because this stage occurs at essentially identical core masses, the maximum luminosity of the RGB is almost the same, independent of the initial mass and chemical composition of the star. The duration of the RGB phase depends on stellar mass going from about 2.0×10^9 yr for a typical 0.7 $M_\odot$ star to about 2.7×10^8 yr for a typical 1.7 $M_\odot$ star.

The evolution of stars lighter than M_{HeF} but heavier than M_{con} is basically similar to the above scheme, although toward the upper mass end it reflects in many respects the evolution of intermediate-mass stars.

In intermediate- and high-mass stars, the main sequence core H-burning phase is characterized by the formation of a convective core, a steady increase in luminosity and radius, and a decrease of the T_{eff}. The size of the convective core, which is customarily fixed by the classical Schwarzschild (1958) criterion ($\nabla_R = \nabla_A$, with the usual meaning of the symbols), increases with stellar mass, whereas the duration of the core H-burning phase decreases with increasing mass owing to the overwhelming effect of the increasing luminosity. The main sequence core H-burning lifetime goes from several 10^8 yr to a few 10^6 yr as the mass of the star increases from about 2 $M_\odot$ to 100 $M_\odot$. Massive stars may be affected by semiconvective instability (thereinafter the H-semiconvection) and mass loss by stellar wind. Semiconvection has long been the characterizing feature of the structure of massive stars evolved at constant mass, whereas to date the most salient signature of the evolution of massive stars is the occurrence of mass loss by stellar wind (see Chiosi & Maeder 1986).

After exhausting central hydrogen while on the main sequence, intermediate- and high-mass stars up to say 15 $M_\odot$ (the evolution is more complicated for the most massive ones) evolve rapidly to the red giant (supergiant) region, burning hydrogen in a thin shell above a rapidly contracting and heating core, composed essentially of helium. As they approach the Hayashi line, a convective envelope develops whose base extends inward until it reaches layers in which hydrogen has been converted into helium and carbon into nitrogen via the CNO cycle. As a consequence, the surface abundance of those elements varies in a detectable way (first dredge-up). H-burning in the shell not only provides the bulk of the stellar luminosity but also adds matter to the H-exhausted core which continues to grow. When temperature and density in the core reach suitable values, helium is ignited.

The question as to why stars become red giants has been debated for many years without a satisfactory answer. Renzini (1984) identifies the physical cause for the rapid expansion of the envelope to red giant dimensions in a thermal instability in the envelope, which is primarily determined by the derivatives of the opacity in the middle temperature region (see also Iben & Renzini 1984). Applegate (1988) finds that a radiative envelope in which a Kramers' opacity law holds cannot transport a luminosity larger than a critical value. He argues that the transition to red giant structure is triggered by the star's luminosity exceeding the critical value. Weiss (1989a) reanalysing the criterion introduced by Renzini (1984) concludes that the opacity is not the main cause. Renzini *et al.* (1992) consider the envelope expansion caused by its thermal instability, whereas according to Iben (1993) there is no simple physical explanation to why stars become red giants. In contrast, Renzini & Ritossa (1994) with numerical experiments document the physical nature of the thermal instability causing the envelope expansion (but for low mass stars). The red giant problem still exists.

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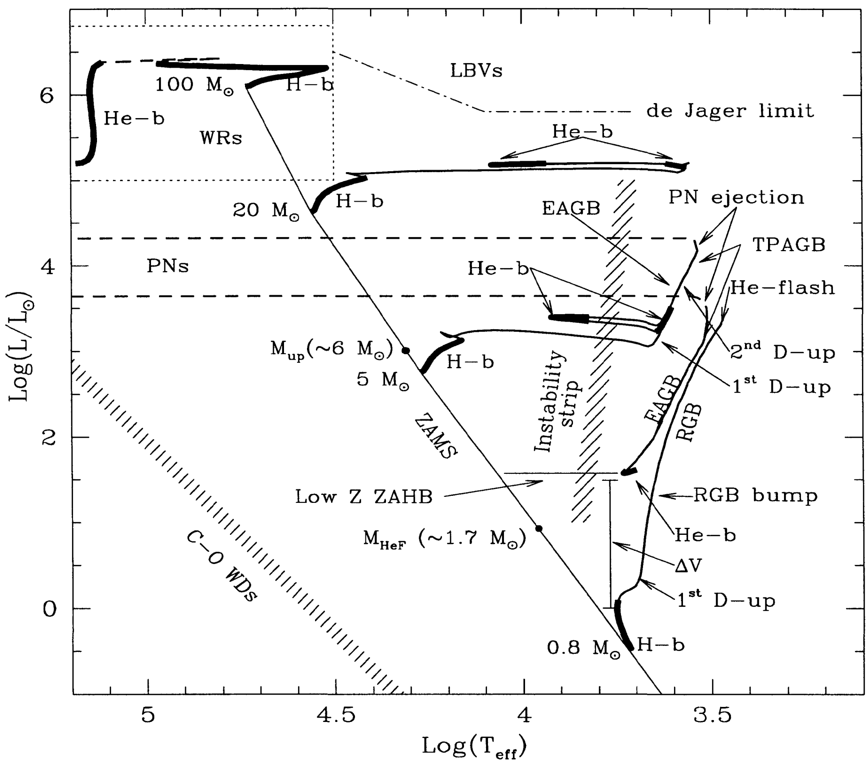

FIGURE 1. The evolutionary paths in the HRD of model stars of composition [Z=0.008, Y=0.25] and of initial mass 0.8 $M_\odot$, 5 $M_\odot$, 20 $M_\odot$, and 100 $M_\odot$. The models are calculated with the overshoot scheme for central convection. M_{HeF} and M_{up} are the masses separating low-mass stars from intermediate-mass stars, and the latter from the massive ones, respectively. For low- and intermediate-mass stars the tracks go from the zero-age main sequence (ZAMS) to the end of the asymptotic giant branch (AGB) phase, whereas for the massive stars they reach the stage of C-ignition in the core. Massive stars include the effect of mass loss by stellar wind. H-b and He-b stand for core H- and He-burning, respectively. He-flash indicates the stage of violent ignition of central He-burning in low-mass stars at the tip of the red giant branch (RGB). The main episodes of external mixing (1^{st} and 2^{nd} dredge-up) are indicated by 1^{st} D-up and 2^{nd} D-up, respectively. The AGB phase is separated into early stages (E-AGB) and thermally pulsing regime (TP-AGB) of the He-burning shell. For low- and intermediate-mass stars we show the stage of planetary nebula (PN) ejection, the region where PN stars are observed, and the white dwarf (WD) cooling sequence. The horizontal line labeled ZAHB indicates the locus of the zero-age horizontal branch – core He-burning models – of low-mass stars with composition typical of globular clusters. The shaded vertical band shows the instability strip of Cepheid and RR Lyrae stars. In the region of massive stars, we show the de Jager limit, the location of the blue luminous variables (LBVs) and Wolf-Rayet stars (WRs). Finally, the thick portions of the tracks indicate the stages of slow evolution, where the majority of stars are observed.

2.3. *Core He-Burning Phase*

The development of the He-flash at the top of the RGB has been the subject of many quasi static as well as dynamical studies aimed at understanding whether the violent burning may acquire explosive characteristics or induce some sort of mixing (see the detailed discussion by Iben & Renzini 1984). Arguments exist for excluding both the total disruption of the star (type II-like supernova) and a substantial mixing between the inner core and the outer envelope (see Renzini & Fusi-Pecci 1988). In fact, type II-like supernovae are not seen in elliptical galaxies but post core He-flash stars do exist (HB and AGB), whereas mixing and consequent dredge-up of carbon would produce a kind of red HB star that is not observed. Following He-flash at the termination of the RGB, stars lighter than M_{HeF} quiescently burn helium in a convective core. Their position on the HB depends on several factors, among which the metallicity and the mass of the H-rich envelope dominate, the latter reduced by mass loss from the red giant precursor. For metal-rich stars, the core He-burning is confined to a narrow region or clump near the RGB about 3 magnitudes below the RGB tip, whereas for metal-poor stars the evolution covers a much broader range of T_{eff}s to the blue of the RGB at approximately the same luminosity as for the metal-rich ones. Under favorable circumstances (sufficiently low metal content or high enough mass loss for high metallicity stars) the HB can intersect the instability strip, giving rise to the RR Lyrae pulsators. The luminosity of the HB stars is determined primarily by the composition (helium abundance) of their main sequence progenitors. The nearly constant luminosity and duration of the He-burning phase (approximately 10^8 yr) reflect the convergency of precursor stars of different initial mass toward a common value for helium core mass (M_{He}). However, as the contribution to the luminosity by the H-burning shell may depend on the mass of the envelope, blue HB models can be less luminous.

The occurrence of mass loss from RGB stars cannot yet be derived from a satisfactory theory, but is basically justified by the observations (e.g. Renzini 1977; Chiosi 1986; Chiosi *et al.* 1992, and below). Since the amount of mass lost by a star depends both on the mean mass-loss rate, customarily expressed by empirical relationships as a function of the stellar parameters (e.g. Reimers 1975), and the duration of the phase in which mass loss is supposed to occur, the observational rates and lifetimes along the RGB are such that mass loss plays an important role only in stars with mass smaller than about 1 $M_\odot$, hence typical of old globular clusters (see Iben 1974, Renzini 1977, and Iben & Renzini 1983 for details). All low-mass stars more massive than about 1 $M_\odot$ remain in the clump for the entire core He-burning phase.

In intermediate- and high-mass stars, core He-burning ignites in nondegenerate conditions as soon as the central temperature and density are approximately equal to 10^8 K and 10^4 g cm^{-3}, respectively. This requires a minimum core mass of 0.33 $M_\odot$. Since M_{He} increases with the initial mass of the star because of the larger convective core on the main sequence, the mean luminosity of core He-burning phase increases with stellar mass. During the core He-burning phase, hydrogen continues to burn in a shell at about the same rate as it did during the main sequence phase. The rate at which helium is burnt in the convective core determines the rate at which the star evolves. Typically, the lifetime in the core He-burning stage is about 20 to 30% of the main sequence lifetime, being longer in models of smaller mass.

The slow evolution during core He-burning of intermediate-mass stars takes place in two distinct regions of the HRD, a first near the Hayashi line and a second at higher T_{eff}s and luminosities. The early stages of core He-burning take place in the first region. Subsequently, when the energy released by the burning core (which is increasing) equals the energy released by the H-burning shell (which is decreasing), a rapid contraction of the envelope readjusts the outer layers from convective to radiative and the star moves to the second region, where the remaining part of the core He-burning phase occurs. This causes the blue loops. The precise modeling and lifetime of the second phase depend on the stellar mass, chemical composition, nuclear reaction rates [$^{12}C(\alpha,\gamma)^{16}O$ in particular], extension of the convective core, opacity, mass loss along the RGB, inward penetration of the outer convection during the RGB stages (first dredge-up), and other physical details. For any choice of composition, as the stellar mass decreases, the location of the blue loop region moves toward the Hayashi line, eventually merging with the red giant region. Thus, for an assigned chemical composition, core He-burning breaks into two bands, one roughly corresponding to the locus of the Hayashi line or red giant stars, and another that breaks off the red giant band at low luminosities and moves toward the blue with increasing luminosity (the so-called blue band). The mean slope of this band is determined by a complicated interplay among the above physical factors which cannot be established a priori. The blue band of the core He-burning models may intersect the instability strip of the Cepheid stars.

Finally, the location of the core He-burning phase of stars more massive than say 15 $M_\odot$ is highly uncertain because it is dominated entirely by the effect of mass loss and convective overshoot (see Chiosi & Maeder 1986). These stars will be discussed in more detail below.

The core He-burning phase of intermediate-mass stars (toward the lower mass end) and low-mass stars on the HB is known to be affected by two types of convective instability: in early stages by a semiconvective mixing similar to that encountered by massive stars, and in late stages by the so-called breathing convection. They will be examined in more detail later on. Suffice it to recall that semiconvection prolongs the He-burning lifetime by approximately a factor of two in low-mass stars, i.e. for stars on the HB, whereas it has a negligible effect in intermediate-mass stars. Breathing convection determines a moderate increase in the lifetime (about 20%), whereas it gives origin to much larger C-O cores in all stars of this mass range.

The result of core He-burning, which turns helium into carbon, oxygen, and traces of heavier species, is the formation of a C-O core whose dimensions are determined by M_{He} and, once again, on the physical model adopted to describe central convection and its efficiency. Low-mass stars form C-O cores of approximately equal mass, whereas all other stars build C-O cores whose sizes increase with stellar mass.

After core He-exhaustion, the structure of the stars is composed of a C-O core, a He-burning shell, and an H-rich envelope at the base of which an H-burning shell is active. However, in massive stars, mass loss by stellar wind may be so strong at this stage that the entire envelope is lost even during the completion of the core He-burning phase (see below).

2.4. *Later Evolutionary Phases*

From the point of view of understanding the HRD, the evolutionary phases beyond the core He-burning of stars more massive than M_{up} are scarcely relevant because of their very short lifetime, hence low probability of detection, were it not for the final supernova explosion. Therefore, their evolution will not be described here (see Chiosi 1986; Woosley 1986, 1988; and Woosley & Weaver 1986).

Following the exhaustion of central helium, low- and intermediate-mass stars evolve through the AGB phase. The AGB phase is separated into two parts: the early AGB or E-AGB, which lasts until the H-burning shell is re-ignited (see below), and the thermally pulsing AGB or TP-AGB (see below), which lasts until the H-rich envelope is lost via a normal giant wind (low-mass progenitors) or via a "superwind" (intermediate-mass progenitors).

As the abundance of helium in central regions goes to zero, the He-exhausted core contracts and heats up while the H-rich envelope expands and cools. Cooling in the layers external to the C-O core is so effective that the H-burning shell extinguishes. In the HRD the stars evolve running almost parallel to the RGB, and once again the base of the convective envelope penetrates inward. According to Iben & Renzini (1983) there is a limiting mass (4.6 $M_\odot$ for solar composition) above which external convection eventually reaches layers processed by the CNO cycle. This means that fresh helium and fresh nitrogen are brought to the surface (second dredge-up). Eventually, the expansion of the envelope is halted by its own cooling and the envelope re-contracts, the luminosity decreases, and matter at the base of the convective envelope heats up. Ultimately, the H-burning shell is re-ignited, forcing the envelope convection to move outward in mass ahead of the H-burning shell. This terminates the E-AGB. In the meantime, the matter in the C-O core reaches such high densities that the electrons there become degenerate. Electron conduction causes the core to become nearly isothermal, while neutrino cooling carries away the gravitational energy liberated by the material added to the core by the outward migration of the He- and H-burning shells. Therefore, the temperature in the core tends to remain close to the temperature in the He-burning shell (about 10^8 K), well below the threshold value for carbon ignition.

Following the re-ignition of the shell H-burning, nuclear burning in the He-shell becomes thermally unstable (for a more detailed discussion see Iben & Renzini 1983, 1984; and Iben 1991). In brief, the nuclear burning does not occur at a steady rate, but the two shells, one H and the other He, alternate as the major source of energy. For 90% of the time the He-burning shell is inactive and the H-burning shell is the major source of energy. However, as the mass of the He-rich zone below the H-burning shell increases, the density and temperature at the base of this zone increase until the rate of energy production by the $3\alpha -^{12}$ C reaction becomes higher than the rate at which energy can be carried out by radiative diffusion. As originally discovered by Schwarzschild & Harm (1965), a thermonuclear runaway occurs. A thin convective layer is generated on the top of the He-burning shell. At first the energy goes into raising the temperature of and expanding the matter in and near the burning zone and the material is pushed away in both directions. Matter at the base of the H-burning region is pushed out and cools to such low temperatures that the H-burning shell is temporarily extinguished. Eventually, matter at the He-burning region begins to cool as it overexpands and the rate of burning there drops dramatically. The convective layer disappears and a steady state is reached in which He-burning occurs quiescently at a slowly decreasing rate as the He-burning shell actually runs out of fuel. This quiescent phase lasts for about 10% of the time elapsing between successive outbursts. The material propelled outward falls back and the H-burning shell eventually re-ignites.

During this phase, material processed into the intershell region can be brought into the outer convective envelope and exposed to the surface. The so-called third dredge-up can then take place. In AGB stars of large C-O core mass (hence with large initial mass) the dredge-up can occur easily. But in AGB stars of small C-O core mass (hence with small initial mass) this is possible only if extra mixing is forced into the intershell region. The goal is achieved either by means of semiconvection induced by the more opaque C-rich material deposited in the intershell region by the tiny convective shell ahead of the flashing He-burning shell or by crude overshoot of convective elements from the convective shell itself. The mechanism of semiconvection has been proposed by Iben & Renzini (1982) following a suggestion by Sackmann (1980). Convective overshoot has subsequently been used by Hollowell (1988) and Hollowell & Iben (1988, 1989). In both discussions C-rich material is deposited in more external layers where it can be easily engulfed by the external convection during the subsequent cycle. This is the basic mechanism to convert an M giant into a carbon star (C star).

In the classical theory of the TP-AGB phase (a deep revision of this scheme will be presented in section 13), the luminosity of the star increases linearly with the mass of the H-exhausted core (Paczynski 1970a,b). Among others, popular relations (slightly different according to the mass range of the progenitor stars) are

$$L = 2.38 \times 10^5 \mu^3 Z_{CNO}^{0.04} \times (M_c^2 - 0.0305 M_c - 0.18021) \quad (2.1)$$

where L is the luminosity in solar units, M_c is the mass of the H-exhausted core (in solar units), μ is the molecular weight, Z_{CNO} is the abundance (by mass) of CNO elements in the envelope. This relation applies to stars with core mass in the range $0.5 M_\odot \leq M_c \leq 0.66 M_\odot$ (Boothroyd & Sackmann 1988a,b).. i.e. in the low mass stars range. For stars with core mass $M_c \geq 0.95 M_\odot$ it is replaced by

$$L = 1.226 \times 10^5 \times \mu^2 \times (M_c - 0.46) M^{0.19} \quad (2.2)$$

originally from Iben & Truran (1978). A linear interpolation is adopted for stars with $0.66 M_\odot \leq M_c \leq 0.95 M_\odot$. This secures that the TP-AGB stars brighten in M_{bol} at a constant rate (see Renzini 1977 and Iben & Renzini 1983 for details).

Given that C-ignition in highly degenerate conditions requires a C-O core mass of 1.4 $M_\odot$ (the Chandrasekhar limit), and considering the effect of mass loss, the minimum initial mass of the star, M_w, for C-ignition to occur is estimated in the range 4 to 6 $M_\odot$, depending on the adopted mass-loss rates, evolutionary lifetimes, and chemical composition (Iben & Renzini 1983). Stars lighter than the above limit will fail C-ignition and, by losing the H-rich envelope will become C-O white dwarfs (WDs) with a modest increase of the C-O core mass during the TP-AGB phase (about 0.1 $M_\odot$). In a very low-mass star (0.8÷1.0) $M_\odot$, the ejection of the envelope may be completed even before the H-burning shell is re-ignited and the thermal pulsing regime begins. Direct observational evidence for the existence of the TP-AGB phase and third dredge-up is given by properties of long-period variables (LPVs) with enhanced strength of the ZrO band. In fact, Zr is formed by s-processing in the convective He-burning shell during a shell flash and is dredged up to the surface (Wood *et al.* 1983).

However, even if intermediate-mass stars with initial mass in the range $M_w \leq M \leq M_{up}$ could experience deflagrating C-ignition, this does not occur for several reasons (Iben 1985, 1991). In short, as we infer from the density of matter in planetary nebula (PN) shells, the estimated outflow rates from OH/IR sources, the several nearby C stars, the paucity of C stars in rich clusters of the LMC (like NGC 1866), and finally the luminosity function of carbon stars in the same galaxy (Reid *et al.* 1990), there must be some fast mechanism, which on a very short time scale (10^3 yr) terminates the TP-AGB phase soon after it begins with a modest increase in the mass of the C-O core with respect to the initial value. The sudden termination of the AGB phase of all intermediate-mass stars has been long attributed to a sort of "superwind" (see Renzini & Voli 1981, and Iben & Renzini 1984), the physical interpretation of which is not yet understood. The manifestation of the superwind could be the OH/IR phenomenon. Estimates of the mass-loss rates from AGB stars and speculations about the physical nature of the superwind have been made by many authors among whom we recall Baud & Habing (1983), Bedijn (1988), van der Veen (1989), and Bowen & Willson (1991). Computations of AGB models including the effect of mass loss are still rare. Recent calculations are by Wood & Vassiliadis (1991) and Vassiliadis & Wood (1993) who identify the superwind in the combined effect of large amplitude radial pulsations and radiation pressure on grains.

The arguments presented above also suggest that the maximum mass of WDs is 1.1 $M_\odot$, considerably lower than the Chandrasekhar mass of 1.4 $M_\odot$, and slightly larger than the value of the C-O core mass at the start of the TP-AGB for a star with initial mass equal to M_{up} (see Iben 1991, and Weidemann 1990).

2.5. *Planetary Nebulae and White Dwarfs*

The main parameters governing the evolution from the AGB to the WD stage through the planetary nebula (PN) phase are the precise stage in a thermal cycle at which the final ejection of the H-rich envelope occurs and the amount of H-rich material which is left on the surface of the remnant at the termination of the ejection phase. Summarizing the results of many authors (Schoenberner 1979, 1981, 1983, 1987; Iben 1984, 1989; Iben & MacDonald 1985, 1986; Iben *et al.* 1983; Renzini 1979, 1982; Wood & Faulkner 1986), three evolutionary schemes are possible: 1. the ejection of the envelope occurs during the quiescent H-burning interpulse phase and the mass dM_{He} of the helium layer between the C-O core and the bottom of the H-rich envelope is "small", i.e. in the range 0.2 to 0.8 dM_H, where dM_H is the mass processed by the H-burning shell between He-shell flashes on the AGB (for a 0.6 $M_\odot$ core, dM_H is about 0.01 $M_\odot$); 2. the ejection occurs in the same stage but dM_{He} is greater than 0.8 dM_H; or 3. the ejection of the H-rich envelope occurs during a He-shell flash or shortly thereafter.

(1) In the first case, following the loss of the envelope, the star evolves blueward at about constant luminosity sustained by the H-burning shell at the base of the residual envelope. The surface temperature gets higher at decreasing mass of the H-rich surface layer. At $T_{eff} \geq 30,000$ K, the flux emitted by the central star ionizes the surrounding nebula and the complex appears as a PN. The time scale is of the order of 10^4 yr for a 0.6 $M_\odot$ star. In this phase, stellar winds from the central stars are known to occur at rates of about $10^{-9} \div 10^{-7}$ $M_\odot$/yr (Perinotto 1983, Cerruti-Sola & Perinotto 1985). When the mass of the H-rich surface layer falls below 10^{-2} dM_H, the H-burning shell extinguishes. The surface layers contract and the luminosity – which is a complicated consequence of gravitational energy release, cooling of non degenerate ions, and neutrino losses (e.g. D'Antona & Mazzitelli 1990) – drops dramatically. Gravitational diffusion becomes so strong that heavy elements sink and hydrogen, if any is left, becomes the dominant element at the surface. Ultimately, the star settles onto the cooling sequence of WDs for the given mass and composition. This model approximates well the characteristics of observed DA-WDs.

(2) In the second case, a final He-shell flash is possible. Following the extinction of the H-burning shell as in the previous case, helium ignites in a shell and the star is pushed back to the tip of the AGB. There the same mechanism that removed the H-rich envelope when the star left the AGB for the first time is likely to operate for a second time, forcing the departure from the AGB. However, in this case the luminosity of the star is sustained by the He-burning shell and departure from the AGB requires that mass loss continues until the residual mass of the H-rich material is less than 10^{-5} $M_\odot$. Evolving to high T_{eff} once again the PN is re-excited, and stellar winds from the central stars cause the loss of all remaining H-rich matter at the surface. The duration of this phase is about three times longer than in the previous case. Eventually He-burning ceases and gravitational sinking of heavy elements makes helium the dominant element at the surface. Finally, the star settles onto the WD sequence. This model nicely corresponds to non-DA-WDs.

(3) In the third case, the H-rich envelope is ejected during a He-shell flash when the intershell region contains the smallest amount of mass. Departed from the AGB, the luminosity, sustained by the He-burning shell, fades to a minimum as the star evolves to higher T_{eff}s. At a certain point, hydrogen is re-ignited and the luminosity increases again at almost constant T_{eff}. The subsequent evolutionary track lies close to the corresponding H-burning track (Iben 1984, Wood & Faulkner 1986). In coincidence with H-re-ignition, a marked slowdown of the evolutionary rate occurs. AGB stars becoming WDs through