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978-0-521-63204-1 - Infinite-Dimensional Dynamical Systems: An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors

James C. Robinson

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