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0521632005 - Theory and Computation of Hydrodynamic Stability

W. O. Criminale, T. L. Jackson and R. D. Joslin

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## THEORY AND COMPUTATION OF HYDRODYNAMIC STABILITY

The study of hydrodynamic stability is fundamental to many subjects, ranging from geophysics and meteorology through to engineering design. This treatise covers both classical and modern aspects of the subject, systematically developing it from the simplest physical problems, then progressing chapter-by-chapter to the most complex, considering linear and nonlinear situations, and analysing temporal and spatial stability. The authors examine each problem both analytically and numerically with many chapters having an appendix outlining appropriate numerical techniques. All relevant fluid flows are treated, including those where the fluid may be compressible, or those from geophysics, or those that require salient geometries for description. Details of initial-value problems are explored equally with those of stability. The text is enriched with many exercises, copious illustrations and an extensive bibliography, and the result is a book that can be used with courses on hydrodynamic stability or as an authoritative reference for researchers.

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# THEORY AND COMPUTATION OF HYDRODYNAMIC STABILITY

W. O. CRIMINALE, T. L. JACKSON, R. D. JOSLIN



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## Preface

The subject of hydrodynamic stability or stability of fluid flow is one that is most important in the fields of aerodynamics, hydromechanics, combustion, oceanography, atmospheric sciences, astrophysics, and biology. Laminar or organized flow is the exception rather than the rule to fluid motion. As a result, exactly what may be the reasons or causes for the breakdown of laminar flow has been a central issue in fluid mechanics for well over a hundred years. And, even with progress, it remains a salient question for there is yet to be a definitive means for prediction. The needs for such understanding are sought in a wide and diverse list of fluid motions because the stability or instability mechanisms determine, to a great extent, the performance of a system. For example, the under prediction of the laminar to turbulent transitional region on aircraft – that is due to hydrodynamic instabilities – would lead to an underestimation of a vehicle's propulsion system and ultimately result in an infeasible engineering design. There are numerous such examples.

The seeds for the writing of this book were sown when one of us (WOC) was contacted by two friends, namely Philip Drazin and David Crighton with the suggestion that it was perhaps time for a new treatise devoted to the subject of stability of fluid motion. A subsequent review was taken by asking many colleagues as to their assessment of this thought and, if this was positive, what should a new writing of this subject entail? The response was enthusiastic and revealed three major requirements: (i) a complete updating of all aspects of the field; (ii) the presentation should provide both analytical and numerical means for solution of any problem posed; (iii) the scope of the treatment should cover the full range of the dynamics, ranging from the transient to asymptotic behavior as well as linear and nonlinear formulations. Then, since the computer is now a major tool, the last need suggested that direct numerical simulation (DNS) must be included as well.

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This challenge was accepted and with intensive collaboration, we have attempted to meet these goals. All prototype flows are considered whether confined (Chapter 3), semi confined (Chapter 3), in the absence of boundaries (Chapter 2) and both parallel, almost parallel or flows with curved stream lines (Chapter 6). In addition, the topics of spatial versus temporal stability (Chapter 4), compressible (Chapter 5) as well as incompressible fluids, geophysical flows (Chapter 7), transition and receptivity (Chapter 10), and optimization and control of flows (Chapter 12) are given full attention. Also, specific initial-value problems (Chapter 8) would be examined as well as the question of stability. In every case, the basics are developed with the physics and the mathematical needs (Chapters 1, 2) with emphasis on numerical methods for solution. To this end, in formulating the organization of the book it was decided that it would be beneficial if, at the end of each chapter that dealt with a specific topic, in addition to exercises for illustration, an appendix, when appropriate, would be attached that provided a numerical basis for that particular area of need. The reader would then be able to develop their own code. Nonlinear stability (Chapter 9) and direct numerical simulation, i.e., DNS (Chapter 11) are supplemented with a review of what is known from experiments (Chapter 13).

The book can easily be used as a text for either an upper level undergraduate or graduate course for this subject. For those who are already knowledgeable, we hope that the book will be a welcome and useful reference.

There are many friends who have helped us with the formulation and writing. Indeed, all have given us both criticism and advice when needed. Particular recognition should be given to Richard DiPrima, who was the mentor of one of us (TLJ) and was a person who provided more than a rationale to be engaged in the field of hydrodynamic stability with his teaching, expertise, and major contributions to the subject. In a similar manner, Robert Betchov provided the initial impetus for another (WOC). More recently, M. Gaster, C. E. Grosch, F. Hu, G. L. Lasseigne, L. Massa and P. J. Schmid have made their time available so that our writing would benefit and the contents be made to fit our goal. To each, we extend our sincere thanks. And, to the late Robert Betchov, Dick DiPrima, David Crighton, and Philip Drazin, a firm note of gratitude. The passing of our colleagues is a loss. Finally, we have had assistance from some who have helped with technical needs. In particular, Frances Chen, Michael Campbell, and Peter Blossey should be cited.