

Cambridge University Press

978-0-521-63193-8 - Holomorphic Spaces

Edited by Sheldon Axler, John E. McCarthy and Donald Sarason

Frontmatter

[More information](#)

The term “holomorphic spaces” is short for “spaces of holomorphic functions.” It refers not so much to a branch of mathematics as to a common thread running through much of modern analysis—through functional analysis, operator theory, harmonic analysis, and, of course, complex analysis.

This is a collection of expository articles arising from MSRI’s Fall 1995 program on holomorphic spaces. The opening article, by Donald Sarason, gives an overview of several aspects of the subject. The remaining articles, while more specialized, are nevertheless designed in varying degrees to be accessible to the nonexpert; some are minicourses in themselves. A range of topics is addressed:

- Bergman spaces (Hedenmalm, Stroethoff)
- Hankel operators in various guises (Gorkin, Peller, Rochberg, Saccone)
- the Dirichlet space (Wu)
- subnormal operators (Conway, Yang)
- operator models, interpolation problems, systems theory (Alpay, Dijksma, Dym, Kheifets, Nikolski, Rovnyak, Sadosky, de Snoo, Vasyunin, Young)

The concluding article, by Victor Vinnikov, describes an approach to certain commuting families of nonselfadjoint operators in which operator theory is linked with algebraic geometry.

Cambridge University Press

978-0-521-63193-8 - Holomorphic Spaces

Edited by Sheldon Axler, John E. McCarthy and Donald Sarason

Frontmatter

[More information](#)

**Mathematical Sciences Research Institute
Publications**

33

Holomorphic Spaces

Cambridge University Press

978-0-521-63193-8 - Holomorphic Spaces

Edited by Sheldon Axler, John E. McCarthy and Donald Sarason

Frontmatter

[More information](#)

Mathematical Sciences Research Institute Publications

- Volume 1* Freed and Uhlenbeck: *Instantons and Four-Manifolds*, second edition
- Volume 2* Chern (editor): *Seminar on Nonlinear Partial Differential Equations*
- Volume 3* Lepowsky, Mandelstam, and Singer (editors): *Vertex Operators in Mathematics and Physics*
- Volume 4* Kac (editor): *Infinite Dimensional Groups with Applications*
- Volume 5* Blackadar: *K-Theory for Operator Algebras*
- Volume 6* Moore (editor): *Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics*
- Volume 7* Chorin and Majda (editors): *Wave Motion: Theory, Modelling, and Computation*
- Volume 8* Gersten (editor): *Essays in Group Theory*
- Volume 9* Moore and Schochet: *Global Analysis on Foliated Spaces*
- Volume 10* Drasin, Earle, Gehring, Kra, and Marden (editors): *Holomorphic Functions and Moduli I*
- Volume 11* Drasin, Earle, Gehring, Kra, and Marden (editors): *Holomorphic Functions and Moduli II*
- Volume 12* Ni, Peletier, and Serrin (editors): *Nonlinear Diffusion Equations and Their Equilibrium States I*
- Volume 13* Ni, Peletier, and Serrin (editors): *Nonlinear Diffusion Equations and Their Equilibrium States II*
- Volume 14* Goodman, de la Harpe, and Jones: *Coxeter Graphs and Towers of Algebras*
- Volume 15* Hochster, Huneke, and Sally (editors): *Commutative Algebra*
- Volume 16* Ihara, Ribet, and Serre (editors): *Galois Groups over \mathbb{Q}*
- Volume 17* Concus, Finn, and Hoffman (editors): *Geometric Analysis and Computer Graphics*
- Volume 18* Bryant, Chern, Gardner, Goldschmidt, and Griffiths: *Exterior Differential Systems*
- Volume 19* Alperin (editor): *Arboreal Group Theory*
- Volume 20* Dazord and Weinstein (editors): *Symplectic Geometry, Groupoids, and Integrable Systems*
- Volume 21* Moschovakis (editor): *Logic from Computer Science*
- Volume 22* Ratiu (editor): *The Geometry of Hamiltonian Systems*
- Volume 23* Baumslag and Miller (editors): *Algorithms and Classification in Combinatorial Group Theory*
- Volume 24* Montgomery and Small (editors): *Noncommutative Rings*
- Volume 25* Akbulut and King: *Topology of Real Algebraic Sets*
- Volume 26* Judah, Just, and Woodin (editors): *Set Theory of the Continuum*
- Volume 27* Carlsson, Cohen, Hsiang, and Jones (editors): *Algebraic Topology and Its Applications*
- Volume 28* Clemens and Kollár (editors): *Current Topics in Complex Algebraic Geometry*
- Volume 29* Nowakowski (editor): *Games of No Chance*
- Volume 30* Grove and Petersen (editors): *Comparison Geometry*
- Volume 31* Levy (editor): *Flavors of Geometry*
- Volume 32* Cecil and Chern (editors): *Tight and Taut Submanifolds*

Volumes 1 through 27 are available from Springer-Verlag

Cambridge University Press

978-0-521-63193-8 - Holomorphic Spaces

Edited by Sheldon Axler, John E. McCarthy and Donald Sarason

Frontmatter

[More information](#)

Holomorphic Spaces

Edited by

Sheldon Axler

San Francisco State University

John E. M^cCarthy

Washington University in St. Louis

Donald Sarason

University of California, Berkeley



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-0-521-63193-8 - Holomorphic Spaces
Edited by Sheldon Axler, John E. McCarthy and Donald Sarason
Frontmatter
[More information](#)

Sheldon Axler
Mathematics Department
San Francisco State University
San Francisco, CA 94132
USA

Mathematical Sciences Research
Institute
1000 Centennial Drive
Berkeley, CA 94720

John E. McCarthy
Department of Mathematics
Campus Box 1146
One Brookings Drive
Washington University
St. Louis MO 63130
USA

MSRI Editorial Committee
Hugo Rossi (chair)
Alexandre Chorin
Silvio Levy (series editor)
Jill Mesirov
Robert Osserman
Peter Sarnak

Donald Sarason
Department of Mathematics
University of California
Berkeley, CA 94720-3840
USA

The Mathematical Sciences Research Institute wishes to acknowledge
support by the National Science Foundation.

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521631938

© Mathematical Sciences Research Institute 1998

First published 1998

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-63193-8 hardback
ISBN-10 0-521-63193-9 hardback

Transferred to digital printing 2007

Cambridge University Press has no responsibility for the persistence or accuracy of
email addresses referred to in this publication.

Contents

Preface	ix
Holomorphic Spaces: A Brief and Selective Survey	1
DONALD SARASON	
Recent Progress in the Function Theory of the Bergman Space	35
HÅKAN HEDENMALM	
Harmonic Bergman Spaces	51
KAREL STROETHOFF	
An Excursion into the Theory of Hankel Operators	65
VLADIMIR V. PELLER	
Hankel-Type Operators, Bourgain Algebras, and Uniform Algebras	121
PAMELA GORKIN	
Tight Uniform Algebras	135
SCOTT SACCONI	
Higher-Order Hankel Forms and Commutators	155
RICHARD ROCHBERG	
Function Theory and Operator Theory on the Dirichlet Space	179
ZHIJIAN WU	
Some Open Problems in the Theory of Subnormal Operators	201
JOHN B. CONWAY AND LIMING YANG	
Elements of Spectral Theory in Terms of the Free Function Model	
Part I: Basic Constructions	211
NIKOLAI NIKOLSKI AND VASILY VASYUNIN	
Liftings of Kernels Shift-Invariant in Scattering Systems	303
CORA SADOSKY	
Some Function-Theoretic Issues in Feedback Stabilisation	337
NICHOLAS YOUNG	

Cambridge University Press

978-0-521-63193-8 - Holomorphic Spaces

Edited by Sheldon Axler, John E. McCarthy and Donald Sarason

Frontmatter

[More information](#)

viii

CONTENTS

The Abstract Interpolation Problem and Applications	351
ALEXANDER KHEIFETS	
A Basic Interpolation Problem	381
HARRY DYM	
Reproducing Kernel Pontryagin Spaces	425
DANIEL ALPAY, AAD DIJKSMA, JAMES ROVNYAK, AND HENDRIK S. V. DE SNOO	
Commuting Operators and Function Theory on a Riemann Surface	445
VICTOR VINNIKOV	

Cambridge University Press

978-0-521-63193-8 - Holomorphic Spaces

Edited by Sheldon Axler, John E. McCarthy and Donald Sarason

Frontmatter

[More information](#)

Holomorphic Spaces
MSRI Publications
Volume 33, 1998

Preface

The term “Holomorphic Spaces” is short for “Spaces of Holomorphic Functions.” It refers not so much to a branch of mathematics as to a common thread running through much of modern analysis—through functional analysis, operator theory, harmonic analysis, and, of course, complex analysis.

In the fall of 1995 the Mathematical Sciences Research Institute in Berkeley sponsored the program Holomorphic Spaces. Over forty participants came for periods of two weeks to a full semester; an additional forty or so attended a week-long workshop in October. Spaces of holomorphic functions arise in many contexts. The MSRI program focused predominantly on operator-theoretic aspects of the subject. A series of minicourses formed the program’s centerpiece.

This volume consists of expository articles by participants in the program (plus collaborators, in two cases), including several articles based on minicourses. The opening article, by Donald Sarason, gives an overview of several aspects of the subject. The remaining articles, while more specialized, are nevertheless designed in varying degrees to be accessible to the nonexpert. A range of topics is addressed: Bergman spaces (Hakan Hedenmalm, Karl Stroethoff); Hankel operators in various guises (Vladimir Peller, Pamela Gorkin, Scott Saccone, Richard Rochberg); the Dirichlet space (Zhijian Wu); subnormal operators (John B. Conway and Liming Yang); operator models and related areas, especially interpolation problems and systems theory (Nikolai Nikolski and Vasily Vasyunin, Cora Sadosky, Nicholas Young, Alexander Kheifets, Harry Dym, James Rovnyak and coauthors). The concluding article, by Victor Vinnikov, describes an approach to certain commuting families of nonself-adjoint operators in which operator theory is linked with algebraic geometry.

The program committee, in addition to the editors of this volume, consisted of Joseph Ball, Nikolai Nikolski, Mihai Putinar, and Cora Sadosky. On behalf of all participants, the program committee wishes to thank the staff of MSRI, especially Director William Thurston, Associate Director Tsit-Yuen Lam, Alisa Colloms, and Kim Garrett, for their many efforts in our behalf.

This volume benefited greatly from the expertise of Silvio Levy, the series editor.

The editors