Relativistic Numerical Hydrodynamics

This book presents an overview of the computational framework in which calculations of relativistic hydrodynamics have been developed. It summarizes the jargon and methods used in the field, and provides illustrative applications to real physical systems. The authors explain how to break down the complexities of Einstein's equations and fluid dynamics, stressing the viability of the Euler–Lagrange approach to astrophysical problems. The book contains techniques and algorithms enabling one to build computer simulations of relativistic fluid problems for various astrophysical systems in one, two, and three dimensions. It also shows the reader how to test relativistic hydrodynamics codes.

Suitable for use as a textbook for graduate courses on astrophysical hydrodynamics and relativistic astrophysics, this book also provides a valuable reference for researchers already working in the field.

JAMES WILSON is widely recognized as a pioneer in the field of numerical relativity and hydrodynamics. Most of the techniques currently in active use in the field today were developed by him at one stage or another. He is best known for having first solved the supernova explosion mechanism by delayed neutrino heating, as well as for developing simulations for accreting black holes, black hole and neutron star collisions, supernova jets, and binary neutron stars. In 1994 he was awarded the Marcel Grossman General Relativity Prize for his contributions to the development of the field of numerical relativity. He is also the author of numerous publications in numerical astrophysics.

GRANT MATHEWS is Professor of Theoretical Astrophysics and Cosmology at Notre Dame University, Indiana. He has been working together with Jim Wilson for the past 15 years on the development of techniques for relativistic hydrodynamics in three spatial dimensions. He has published over 200 papers in areas of theoretical and experimental astrophysics, cosmology, and relativity.

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Relativistic Numerical Hydrodynamics

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Dedicated to our loving and patient wives Demetra and Eve

Contents

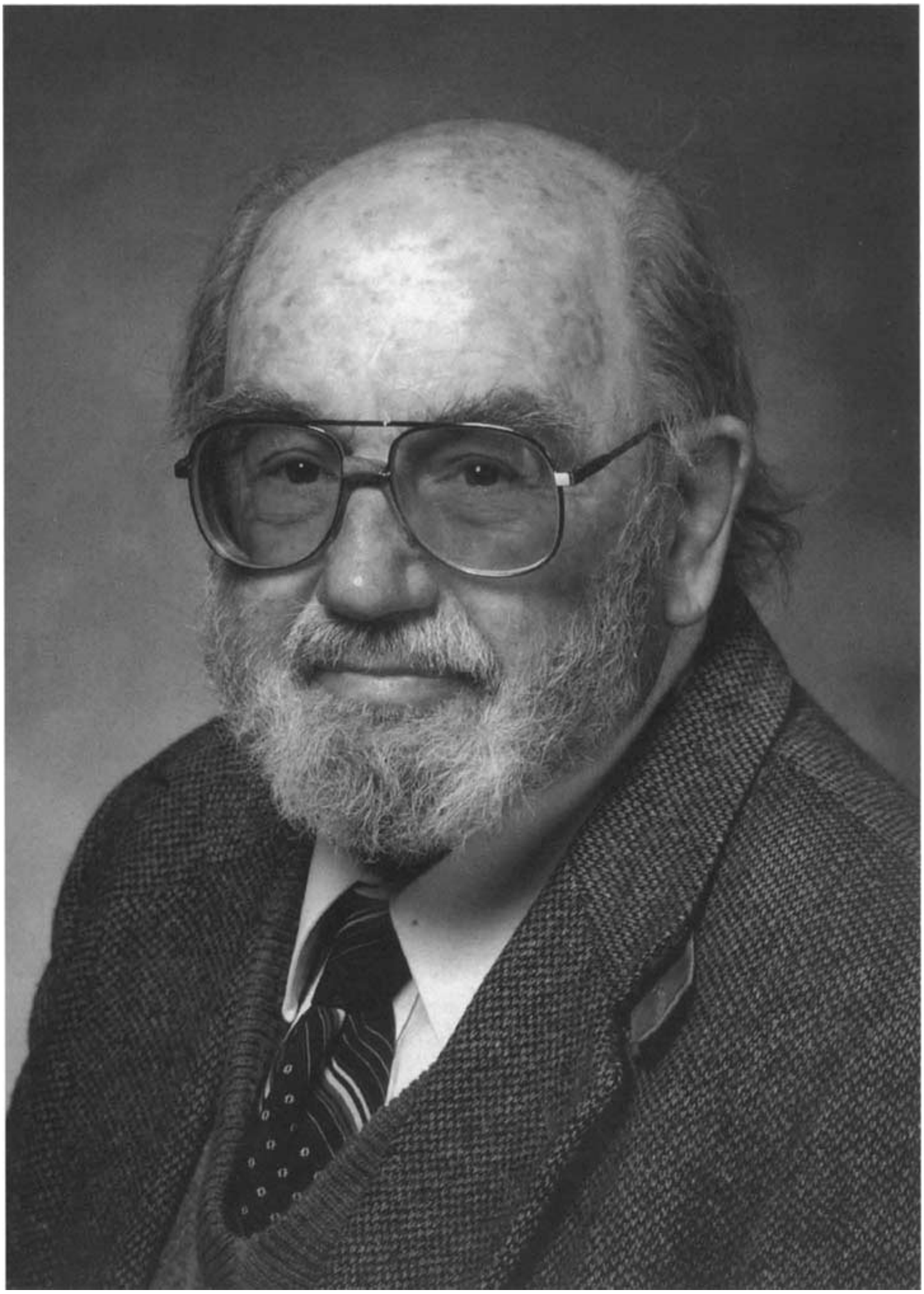
Preface

We are convinced of a genuine need for a monograph describing the many facets and new developments in numerical relativistic hydrodynamics. Such calculations are crucial to several areas of current research in the physics of stellar collapse, supernovae, and black hole formation, as well as the merging of the final orbits of coalescing binary neutron stars. Both problems are only now entering the level of sophistication where three-dimensional relativistic hydrodynamics simulations are both possible and necessary. In the former problem such calculations are crucial to understand the explosion mechanism. In the latter problem, a great deal of interest in such calculations has recently been inspired by the development of next-generation gravity wave detectors to search for such events, and as a possible explanation of the physics underlying observed astrophysical γ-ray bursts.

The field of numerical relativistic hydrodynamics has developed over the past 30 years, but there has not been written a technical text explaining the many techniques relevant to this discipline, many of which are much different than standard general relativity textbook approaches. This book will present such a review of techniques for numerical general relativistic hydrodynamics developed by one of the pioneers of this field over the past three decades.

We begin by developing the equations and differencing schemes for special relativistic hydrodynamics as an introduction to the metric formulation of the problems. Here, the basic numerical techniques and a number of test problems and applications will be discussed.

Following this, the formalism for matter flows in the curved spacetime of general relativity will be presented in the usual (3+1) formalism. With the techniques established, the next chapter will then summarize cosmological applications in one spatial dimension. This will also lead naturally to



a discussion of core-collapse supernovae in spherical symmetry including the many physical complexities due to neutrino interactions and a large range of dynamic timescales.

Next we will describe some important axisymmetric problems such as stellar and black hole rotation, accretion and the head-on collision of two neutron stars. This topic also naturally leads to a discussion of magneto-hydrodynamics and its applications to axisymmetric problems.

The book then finally discusses an application in three spatial dimensions: the hydrodynamics of orbiting neutron stars. This chapter focuses on the development of the conformally flat approximation and techniques for analyzing the gravitational radiation generated by stellar collapse and binary mergers.

This book would best be described as a monograph. That is, it is a summary by experts in the field written for others at a similar level. Nevertheless, enough introductory material has been included that a graduate student or nonexpert can become familiar with the concepts without additional resource material. The main point of this book is to provide a summary of results and techniques for both the expert and nonexpert in one complete text. Some of this material has never been published and only exists in private notes. Most of the available material only exists in a number of journal publications and/or obscure conference proceedings, many of which are no longer in print.

The work described herein is of course the result of the efforts of many knowledgeable collaborators. We would particularly like to acknowledge the important contributions to the general relativistic hydrodynamics work discussed herein from Joan Centrella, Sam Dalhed, Steven Detweiler, Peter Dykema, Charles Evans, Chris Fragile, Hannu Kurki-Suonio, James LeBlanc, Pedro Marronetti, Richard Matzner, Ronald Mayle, Thomas McAbee, Jay Salmonson, and Larry Smarr. We would also like to acknowledge useful input from Peter Anninos and Dinshaw Balsara, along with help from Heidi Grantham in the preparation of some of the figures.