
Contents

INTRODUCTION	viii
I. FIRST ORDER REASONING	1
1.1 Substitution	2
1.2 Denotation and Description	11
1.3 Functions and Relations	20
1.4 Direct Reasoning	25
1.5 Proof Boxes	30
1.6 Formal and Idiomatic Proof	35
1.7 Automated Deduction	44
1.8 Classical and Intuitionistic Logic	52
Exercises I	60
II. TYPES AND INDUCTION	65
2.1 Constructing the Number Systems	67
2.2 Sets (Zermelo Type Theory)	72
2.3 Sums, Products and Function-Types	81
2.4 Propositions as Types	87
2.5 Induction and Recursion	95
2.6 Constructions with Well Founded Relations	102
2.7 Lists and Structural Induction	106
2.8 Higher Order Logic	112
Exercises II	119

vi	<i>Contents</i>	
III.	POSETS AND LATTICES	125
	3.1 Posets and Monotone Functions	126
	3.2 Meets, Joins and Lattices	131
	3.3 Fixed Points and Partial Functions	136
	3.4 Domains	140
	3.5 Products and Function-Spaces	144
	3.6 Adjunctions	151
	3.7 Closure Conditions and Induction	156
	3.8 Modalities and Galois Connections	161
	3.9 Constructions with Closure Conditions	169
	Exercises III	175
IV.	CARTESIAN CLOSED CATEGORIES	183
	4.1 Categories	184
	4.2 Actions and Sketches	190
	4.3 Categories for Formal Languages	197
	4.4 Functors	206
	4.5 A Universal Property: Products	212
	4.6 Algebraic Theories	222
	4.7 Interpretation of the Lambda Calculus	228
	4.8 Natural Transformations	235
	Exercises IV	244
V.	LIMITS AND COLIMITS	250
	5.1 Pullbacks and Equalisers	251
	5.2 Subobjects	255
	5.3 Partial and Conditional Programs	261
	5.4 Coproducts and Pushouts	268
	5.5 Extensive Categories	274
	5.6 Kernels, Quotients and Coequalisers	280
	5.7 Factorisation Systems	286
	5.8 Regular Categories	292
	Exercises V	298

Contents vii

VI.	STRUCTURAL RECURSION	306
	6.1 Free Algebras for Free Theories	307
	6.2 Well Formed Formulae	315
	6.3 The General Recursion Theorem	322
	6.4 Tail Recursion and Loop Programs	329
	6.5 Unification	339
	6.6 Finiteness	343
	6.7 The Ordinals	352
	Exercises VI	360
VII.	ADJUNCTIONS	367
	7.1 Examples of Universal Constructions	368
	7.2 Adjunctions	375
	7.3 General Limits and Colimits	382
	7.4 Finding Limits and Free Algebras	391
	7.5 Monads	397
	7.6 From Semantics to Syntax	404
	7.7 Gluing and Completeness	412
	Exercises VII	420
VIII.	ALGEBRA WITH DEPENDENT TYPES	426
	8.1 The Language	429
	8.2 The Category of Contexts	437
	8.3 Display Categories and Equality Types	449
	8.4 Interpretation	456
	Exercises VIII	467
IX.	THE QUANTIFIERS	469
	9.1 The Predicate Convention	470
	9.2 Indexed and Fibred Categories	476
	9.3 Sums and Existential Quantification	487
	9.4 Dependent Products	495
	9.5 Comprehension and Powerset	506
	9.6 Universes	512
	Exercises IX	523
	BIBLIOGRAPHY	530
	INDEX	553