Mathematical Explorations with MATLAB

This book is about the kind of mathematics usually encountered in first year university courses. A key feature of the book is that this mathematics is explored in depth using the popular and powerful package MATLAB. The emphasis is on understanding and investigating the mathematics, and putting it into practice in a wide variety of modelling situations. In the process, the reader will gain some fluency with MATLAB, no starting knowledge of the package being assumed. The range of material is wide: matrices, whole numbers, complex numbers, geometry of curves and families of lines, data analysis, random numbers and simulations, and differential equations form the basic mathematics. This is applied to a large number of investigations and modelling problems, from sequences of real numbers to cafeteria queues, from card shuffling to models of fish growth. All extras to the standard MATLAB package are supplied on the World Wide Web.

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Preface

Mathematics and its practitioners have come a long way since the days of drawing polygons in the sand with a stick. Although this cannot be said of all our degree courses, there is nevertheless an increasing realisation in higher mathematics education that current computing technology can open new doors for students and tutors alike. This book arose out of a largely successful attempt to complement traditional mathematical courses with one which took this opportunity seriously.

First year students at a UK university are expected to acquire a wide range of mathematical skills—the ability to argue logically, absorb new concepts, calculate accurately, translate everyday problems into appropriate mathematical language, construct mathematical models and to assess the approximations made. We chose to use the popular and powerful computer package MATLAB® to help promote some of these skills. It provided a convenient way to help students understand things graphically, to see the wood rather than the trees in complex problems and to give access to more realistic modelling situations.

We chose MATLAB rather than one of the increasingly sophisticated and algebraically based packages because of the very gently sloped learning curve involved. MATLAB allows the student to graduate smoothly from the functionality of a hand calculator, through increasing use of powerful numerical and graphical facilities towards a high level programming capability. The latter point was considered a bonus in that it provided a possible access route to programming for students with no prior computer background. At the very least, students with no keyboard skills at all can acquire a degree of familiarity with an essential modern tool, the computer.

The course, and this book, were designed for students coming to grips with a typical first year honours mathematics course at a UK university.

In our case, students had already completed the first semester of core units and so already had a basic knowledge of calculus, complex numbers, vectors and matrices. In the book, we assume that the reader has a reasonable level of skill in calculus but only limited familiarity with the other topics. The typical student will be in the process of extending this base to include some selection of topics such as elementary statistics, mechanics, linear algebra, number theory, differential equations, Fourier series and so on. The book is thus intended to help motivate new topics and to build on old ones.

Like Gaul, the book is divided into three parts. Part one comprises a very elementary 'hands-on' introduction to the features of MATLAB followed by a series of methods chapters. In these the reader is taken through a range of mathematical ideas and given 'on the job' training in those MATLAB techniques which are expected to be of particular value in the ensuing project chapters. Thus all the standard programming structures and MATLAB commands are introduced through work on: matrices; whole numbers and elementary number theory; graphing plane curves; data fitting and approximations to functions using least squares techniques; simulation of random distributions; and ordinary differential equations. In this way, the student learning how to use MATLAB is taken through mathematics which is (or should be!) interesting for its own sake.

Part two contains a variety of projects, termed 'Investigations', which build on the earlier ideas. Matrices are applied in the context of magic squares, permutations and the solution of linear systems; manipulation of whole numbers is applied to greatest common divisors of random sets of numbers, primality testing and card shuffling; approximation techniques are applied to solution of nonlinear equations and interpolations; and so on. In each case, an exploratory attitude is encouraged, backed up with plenty of explicit exercises, both purely computational and more mathematical in nature.

Finally, Part three contains a number of 'Modelling Projects' in which the reader is invited to employ some of the skills developed in Part one. By its nature, mathematical modelling is a rather open-ended process and requires a certain degree of mathematical maturity that a first year student may not yet have attained. Nevertheless, we feel that the availability of techniques to which MATLAB gives access, and the very great importance of modelling as an applied mathematical skill, mean that this is an opportunity not to be missed. In practice, we have found that students cope well with these challenges.

At Liverpool, we required students to work through the preliminary material (Part one, taking six weeks), and allowed them to choose a total of three projects from Parts two and three, with at least one from each part. Two weeks seemed to be a good time to allow for the completion of one project, so that the whole course was twelve weeks long.

We have of course striven for uniformity in important matters throughout the book. But a discerning reader will detect three different styles in the project work of Parts two and three, providing a measure of variety which we feel is entirely healthy.

We have provided appendices which list MATLAB commands, give some information on symbolic calculations (not used explicitly in the material of the book) and MATLAB resources, and list the available M-files chapter by chapter.

Using the book

The book will prove useful in a number of contexts. Firstly it can be used, as it stands, to deliver a complete course unit. Secondly, the book should prove useful to course designers with slightly differing requirements. In this case the various examples of project work will provide a convenient source of material and stimulate the creation of further material tailored to the local need. Thirdly, the book can serve as a self-contained tutor for the enthusiastic individual who is not following any formal course structure.

In every case, the reader is intended to work through the book while sitting at the computer keyboard, although there are also mathematical exercises to be done off-line. A preliminary skim through this preface and Chapter 1 will help orient the newcomer before plunging in.

Readers who already have some experience with MATLAB might well wish to jump straight into Chapter 2. If in doubt, readers can quickly brush up their skills with the exercises at the end of Chapter 1.

Copies of all the 'M-files' to which the text refers are freely available. Details of how to obtain these are given in the Appendix. Partial solutions and hints are available to course tutors in electronic form on request from the publisher.

MATLAB is available on a wide variety of platforms. For definiteness, the book assumes the reader has access to MATLAB within a Microsoft Windows environment. Should this not be the case, readers may experience some small inconvenience in the early stages while adapting file-handling and editing instructions to suit their own installation, but the M-files should all run correctly and the material of the book itself is

platform independent. Course providers might wish to make available a brief summary of key points where the local reader might otherwise go astray.

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Liverpool, October 1998

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