

Cambridge University Press

978-0-521-63078-8 - Mathematical Explorations with MATLAB

Ke Chen, Peter Giblin and Alan Irving

Frontmatter

[More information](#)

---

## Mathematical Explorations with MATLAB

This book is about the kind of mathematics usually encountered in first year university courses. A key feature of the book is that this mathematics is explored in depth using the popular and powerful package MATLAB. The emphasis is on understanding and investigating the mathematics, and putting it into practice in a wide variety of modelling situations. In the process, the reader will gain some fluency with MATLAB, no starting knowledge of the package being assumed. The range of material is wide: matrices, whole numbers, complex numbers, geometry of curves and families of lines, data analysis, random numbers and simulations, and differential equations form the basic mathematics. This is applied to a large number of investigations and modelling problems, from sequences of real numbers to cafeteria queues, from card shuffling to models of fish growth. All extras to the standard MATLAB package are supplied on the World Wide Web.

All three authors hold positions at the University of Liverpool.

Ke Chen is Lecturer in Mathematical Sciences, Peter Giblin is Reader in Mathematics, and Alan Irving is Reader in Theoretical Physics.

Cambridge University Press

978-0-521-63078-8 - Mathematical Explorations with MATLAB

Ke Chen, Peter Giblin and Alan Irving

Frontmatter

[More information](#)

---

Cambridge University Press  
978-0-521-63078-8 - Mathematical Explorations with MATLAB  
Ke Chen, Peter Giblin and Alan Irving  
Frontmatter  
[More information](#)

---

# Mathematical Explorations with MATLAB

KE CHEN, PETER GIBLIN, ALAN IRVING



Cambridge University Press

978-0-521-63078-8 - Mathematical Explorations with MATLAB

Ke Chen, Peter Giblin and Alan Irving

Frontmatter

[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521630788](http://www.cambridge.org/9780521630788)

© Cambridge University Press 1999

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 1999  
Reprinted 2000

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-63078-8 Hardback  
ISBN 978-0-521-63920-0 Paperback

Cambridge University Press has no responsibility for the persistence or  
accuracy of URLs for external or third-party internet websites referred to in  
this publication, and does not guarantee that any content on such websites is,  
or will remain, accurate or appropriate. Information regarding prices, travel  
timetables, and other factual information given in this work is correct at  
the time of first printing but Cambridge University Press does not guarantee  
the accuracy of such information thereafter.

Contents

<i>Preface</i>	<i>page xi</i>
<b>Part one: Foundations</b>	<b>1</b>
<b>1 Introduction</b>	<b>3</b>
1.1 First steps with MATLAB	3
1.2 Vectors and plots	6
1.3 Creating and editing script files	9
1.4 Getting hardcopy of things	15
Exercises	16
<b>2 Matrices and Complex Numbers</b>	<b>18</b>
2.1 Vectors and matrices	18
2.2 Complex numbers	23
2.3 Population dynamics: the Leslie matrix	24
Exercises	27
<b>3 Whole Numbers</b>	<b>31</b>
3.1 A loop to calculate Fibonacci numbers	31
3.2 A loop with conditionals: the $3n + 1$ or hailstone problem	33
3.3 The euclidean algorithm for greatest common divisors	34
3.4 Fermat's theorem and the power algorithm	36
Exercises	39
3.5 Appendix	41
<b>4 Graphs and Curves</b>	<b>44</b>
4.1 Polynomials	44
4.2 Initial examples of drawing curves	45

vi	<i>Contents</i>	
4.3	Taylor polynomials	47
4.4	Approximations using the function polyfit	49
4.5	The goat problem	50
4.6	Envelopes of lines	51
	Exercises	54
4.7	Appendix	58
<b>5</b>	<b>Representation of Data</b>	<b>60</b>
5.1	Data analysis	60
5.2	Least squares fitting	63
	Exercises	70
5.3	Appendix	71
<b>6</b>	<b>Probability and Random Numbers</b>	<b>75</b>
6.1	Generating random numbers	75
6.2	Random integers	76
6.3	Simulating uniform distributions	78
6.4	Simulating normal distributions	78
6.5	Simulating negative exponential distributions	79
	Exercises	82
6.6	Appendix	84
<b>7</b>	<b>Differential and Difference Equations</b>	<b>88</b>
7.1	Ordinary differential equations (ODEs)	88
7.2	Systems of differential equations	91
7.3	Difference equations	94
	Exercises	96
	<b>Part two: Investigations</b>	<b>99</b>
<b>8</b>	<b>Magic Squares</b>	<b>101</b>
8.1	Introduction	102
8.2	Magic squares size $3 \times 3$	102
8.3	Magic squares size $4 \times 4$	105
8.4	Magic squares size $5 \times 5$ (optional)	107
<b>9</b>	<b>GCDs, Pseudoprimes and Miller's Test</b>	<b>108</b>
A	GCDs of random pairs and triples of numbers	108
B	Pseudoprimes and Miller's test	111

<i>Contents</i>	vii
<b>10 Graphics: Curves and Envelopes</b>	<b>116</b>
A Rose curves and epicycloids	116
B Envelopes	119
C Curves of constant width	122
<b>11 Zigzags and Fast Curves</b>	<b>128</b>
A Spirographs and zigzags	128
B Fast curves	135
<b>12 Sequences of Real Numbers</b>	<b>146</b>
12.1 Möbius sequences	146
12.2 Cobweb diagrams	148
12.3 Möbius functions and powers of matrices	149
A Investigation on Möbius sequences	153
B Attracting cycles	156
C Quadratic and exponential sequences; fixed points	159
<b>13 Newton–Raphson Iteration and Fractals</b>	<b>164</b>
13.1 Introduction	164
13.2 The equation $z^2 + 1 = 0$	165
13.3 General quadratic equations	166
13.4 The cubic equation $z^3 - z = 0$	168
<b>14 Permutations</b>	<b>171</b>
A Cycle decompositions	171
B Card shuffling	177
14.1 Introduction	177
14.2 Ins and outs	179
14.3 Cycles	185
14.4 Rough riffles (ruffles)	185
14.5 Appendix	188
<b>15 Iterations for Nonlinear Equations</b>	<b>189</b>
15.1 1D: Method 1 — Newton–Raphson	190
15.2 1D: Method 2 — Gauss–Jacobi	191
15.3 1D: Convergence analysis	191
15.4 2D: Iterations for nonlinear systems	196
15.5 2D: Contour plot and convergence history	199
Exercises	203

viii	<i>Contents</i>	
<b>16</b>	<b>Matrices and Solution of Linear Systems</b>	<b>207</b>
16.1	Operation counts	208
16.2	Dense linear systems	209
16.3	The iterative refinement algorithm	212
16.4	A perturbation analysis for $Ax = b$	213
16.5	Sparse matrices, graph ordering and permutations	214
	Exercises	217
<b>17</b>	<b>Function Interpolations and Approximation</b>	<b>221</b>
17.1	1D: Introduction	222
17.2	The 1D example M-file <code>intdemo1.m</code>	223
17.3	1D data fitting	224
17.4	How accurate is my approximation?	227
17.5	Introduction to multi-variable approximation	228
17.6	The 2D M-file <code>intdemo2.m</code>	229
17.7	Contour plots, 3D plots and slicing	229
17.8	The ‘\’ global method	233
17.9	The piecewise method	234
17.10	Comparison of approximations	234
	Exercises	236
<b>18</b>	<b>Ordinary Differential Equations</b>	<b>239</b>
18.1	Strategy	239
	Exercises	240
<b>Part three:</b>	<b>Modelling</b>	<b>243</b>
<b>19</b>	<b>Checkout Queues: Long or Short</b>	<b>245</b>
19.1	Simulating queues	246
19.2	The motorway filling station	250
19.3	The Leo’s cafeteria	251
	Exercises	253
<b>20</b>	<b>Fish Farming</b>	<b>257</b>
20.1	Preliminary look at the problem	258
20.2	Models of fish growth	258
20.3	Designing the Leslie matrix	261
20.4	Fishing strategy	262
	Exercises	263



<i>Contents</i>	<i>ix</i>
<b>21 Epidemics</b>	<b>265</b>
21.1 Preliminary look at some data	266
21.2 The SIR model for the dynamics of an epidemic	267
21.3 Studying the behaviour analytically	268
21.4 Analysing the data	269
Exercises	271
<b>22 Dynamics of Snowboating</b>	<b>273</b>
22.1 Preliminary look at the problem	274
22.2 The equations of motion	275
22.3 Exploring the operating parameters	277
Exercises	278
<b>23 Tides</b>	<b>281</b>
23.1 Preliminary look at the tidal data	282
23.2 Fourier series and methods	282
23.3 Analysis of an electrical signal	284
23.4 Fourier analysis of the tidal data	284
Exercises	285
<b><i>Appendix 1</i> MATLAB Command Summary</b>	<b>286</b>
<b><i>Appendix 2</i> Symbolic Calculations within MATLAB</b>	<b>290</b>
<b><i>Appendix 3</i> List of All M-files Supplied</b>	<b>292</b>
<b><i>Appendix 4</i> How to Get Solution M-files</b>	<b>296</b>
<b><i>Appendix 5</i> Selected MATLAB Resources on the Internet</b>	<b>297</b>
<i>References</i>	299
<i>Index</i>	301

Cambridge University Press

978-0-521-63078-8 - Mathematical Explorations with MATLAB

Ke Chen, Peter Giblin and Alan Irving

Frontmatter

[More information](#)

---

Cambridge University Press

978-0-521-63078-8 - Mathematical Explorations with MATLAB

Ke Chen, Peter Giblin and Alan Irving

Frontmatter

[More information](#)

## Preface

Mathematics and its practitioners have come a long way since the days of drawing polygons in the sand with a stick. Although this cannot be said of all our degree courses, there is nevertheless an increasing realisation in higher mathematics education that current computing technology can open new doors for students and tutors alike. This book arose out of a largely successful attempt to complement traditional mathematical courses with one which took this opportunity seriously.

First year students at a UK university are expected to acquire a wide range of mathematical skills—the ability to argue logically, absorb new concepts, calculate accurately, translate everyday problems into appropriate mathematical language, construct mathematical models and to assess the approximations made. We chose to use the popular and powerful computer package MATLAB<sup>®</sup> to help promote some of these skills. It provided a convenient way to help students understand things graphically, to see the wood rather than the trees in complex problems and to give access to more realistic modelling situations.

We chose MATLAB rather than one of the increasingly sophisticated and algebraically based packages because of the very gently sloped learning curve involved. MATLAB allows the student to graduate smoothly from the functionality of a hand calculator, through increasing use of powerful numerical and graphical facilities towards a high level programming capability. The latter point was considered a bonus in that it provided a possible access route to programming for students with no prior computer background. At the very least, students with no keyboard skills at all can acquire a degree of familiarity with an essential modern tool, the computer.

The course, and this book, were designed for students coming to grips with a typical first year honours mathematics course at a UK university.

Cambridge University Press  
978-0-521-63078-8 - Mathematical Explorations with MATLAB  
Ke Chen, Peter Giblin and Alan Irving  
Frontmatter  
[More information](#)

In our case, students had already completed the first semester of core units and so already had a basic knowledge of calculus, complex numbers, vectors and matrices. In the book, we assume that the reader has a reasonable level of skill in calculus but only limited familiarity with the other topics. The typical student will be in the process of extending this base to include some selection of topics such as elementary statistics, mechanics, linear algebra, number theory, differential equations, Fourier series and so on. The book is thus intended to help motivate new topics and to build on old ones.

Like Gaul, the book is divided into three parts. Part one comprises a very elementary ‘hands-on’ introduction to the features of MATLAB followed by a series of methods chapters. In these the reader is taken through a range of mathematical ideas and given ‘on the job’ training in those MATLAB techniques which are expected to be of particular value in the ensuing project chapters. Thus all the standard programming structures and MATLAB commands are introduced through work on: matrices; whole numbers and elementary number theory; graphing plane curves; data fitting and approximations to functions using least squares techniques; simulation of random distributions; and ordinary differential equations. In this way, the student learning how to use MATLAB is taken through mathematics which is (or should be!) interesting for its own sake.

Part two contains a variety of projects, termed ‘Investigations’, which build on the earlier ideas. Matrices are applied in the context of magic squares, permutations and the solution of linear systems; manipulation of whole numbers is applied to greatest common divisors of random sets of numbers, primality testing and card shuffling; approximation techniques are applied to solution of nonlinear equations and interpolations; and so on. In each case, an exploratory attitude is encouraged, backed up with plenty of explicit exercises, both purely computational and more mathematical in nature.

Finally, Part three contains a number of ‘Modelling Projects’ in which the reader is invited to employ some of the skills developed in Part one. By its nature, mathematical modelling is a rather open-ended process and requires a certain degree of mathematical maturity that a first year student may not yet have attained. Nevertheless, we feel that the availability of techniques to which MATLAB gives access, and the very great importance of modelling as an applied mathematical skill, mean that this is an opportunity not to be missed. In practice, we have found that students cope well with these challenges.

Cambridge University Press  
978-0-521-63078-8 - Mathematical Explorations with MATLAB  
Ke Chen, Peter Giblin and Alan Irving  
Frontmatter  
[More information](#)

*Preface*

xiii

At Liverpool, we required students to work through the preliminary material (Part one, taking six weeks), and allowed them to choose a total of three projects from Parts two and three, with at least one from each part. Two weeks seemed to be a good time to allow for the completion of one project, so that the whole course was twelve weeks long.

We have of course striven for uniformity in important matters throughout the book. But a discerning reader will detect three different styles in the project work of Parts two and three, providing a measure of variety which we feel is entirely healthy.

We have provided appendices which list MATLAB commands, give some information on symbolic calculations (not used explicitly in the material of the book) and MATLAB resources, and list the available M-files chapter by chapter.

**Using the book**

The book will prove useful in a number of contexts. Firstly it can be used, as it stands, to deliver a complete course unit. Secondly, the book should prove useful to course designers with slightly differing requirements. In this case the various examples of project work will provide a convenient source of material and stimulate the creation of further material tailored to the local need. Thirdly, the book can serve as a self-contained tutor for the enthusiastic individual who is not following any formal course structure.

In every case, the reader is intended to work through the book while sitting at the computer keyboard, although there are also mathematical exercises to be done off-line. A preliminary skim through this preface and Chapter 1 will help orient the newcomer before plunging in.

Readers who already have some experience with MATLAB might well wish to jump straight into Chapter 2. If in doubt, readers can quickly brush up their skills with the exercises at the end of Chapter 1.

Copies of all the ‘M-files’ to which the text refers are freely available. Details of how to obtain these are given in the Appendix. Partial solutions and hints are available to course tutors in electronic form on request from the publisher.

MATLAB is available on a wide variety of platforms. For definiteness, the book assumes the reader has access to MATLAB within a Microsoft Windows environment. Should this not be the case, readers may experience some small inconvenience in the early stages while adapting file-handling and editing instructions to suit their own installation, but the M-files should all run correctly and the material of the book itself is

Cambridge University Press  
978-0-521-63078-8 - Mathematical Explorations with MATLAB  
Ke Chen, Peter Giblin and Alan Irving  
Frontmatter  
[More information](#)

---

xiv

*Preface*

platform independent. Course providers might wish to make available a brief summary of key points where the local reader might otherwise go astray.

**Acknowledgments**

We are very grateful to colleagues at Liverpool who have helped to set up this course and have provided input to the material. These are Nigel Backhouse, Eric Edmond, Toby Hall, Neil Kirk, Dick Wait, Neville Waters. We are especially grateful to the students who have taken the course over several years; their efforts have detected possible ambiguities in the project work and helped to make the material more user-friendly — and, we hope, correct — than it was at the beginning. Any remaining faults are of course our responsibility. PG is also grateful to Brown University in Providence for its generous hospitality during part of the writing period of this book, and to the Fulbright Commission for a travel grant.

Liverpool, October 1998

Ke Chen  
Peter Giblin  
Alan Irving

MATLAB<sup>®</sup> is a registered trademark of The MathWorks, Inc.