GENERAL RELATIVITY

Starting with the idea of an event and finishing with a description of the standard big-bang model of the Universe, this textbook provides a clear, concise, and up-to-date introduction to the theory of general relativity, suitable for final-year undergraduate mathematics or physics students. Throughout, the emphasis is on the geometric structure of spacetime, rather than the traditional coordinate-dependent approach. This allows the theory to be pared down and presented in its simplest and most elegant form. Topics covered include flat spacetime (special relativity), Maxwell fields, the energy–momentum tensor, spacetime curvature and gravity, Schwarzschild and Kerr spacetimes, black holes and singularities, and cosmology.

In developing the theory, all physical assumptions are clearly spelled out, and the necessary mathematics is developed along with the physics. Exercises are provided at the end of each chapter and key ideas in the text are illustrated with worked examples. Solutions and hints to selected problems are also provided at the end of the book.

This textbook will enable the student to develop a sound understanding of the theory of general relativity and all the necessary mathematical machinery.

Dr. Ludvigsen received his first Ph.D. from Newcastle University and his second from the University of Pittsburgh. His research at the University of Botswana, Lesotho, and Swaziland led to an Andrew Mellon Fellowship in Pittsburgh, where he worked with the renowned relativist Ted Newman on problems connected with *H*-space and nonlinear gravitons. Dr. Ludvigsen is currently serving as both docent and lecturer at the University of Linköping in Sweden.

GENERAL RELATIVITY

A GEOMETRIC APPROACH

Malcolm Ludvigsen

University of Linköping

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK http: //www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA http: //www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1999

Printed in the United States of America

Typeset in Melior 9.75/13pt. and Eurostile in LaTeX 2_ε [TB]

*A catalog record for this book is available from
the British Library.*

Library of Congress Cataloging-in-Publication Data

Ludvigsen, Malcolm. 1946–
General relativity : a geometric approach / Malcolm Ludvigsen.
p. cm.
Includes bibliographical references and index.
ISBN 0-521-63019-3 (hardbound)
1. General relativity (Physics) 2. Space and time.
3. Geometrodynamics. I. Title.
QC173.6.L83 1999
530.11 – dc21 98-37546
CIP

ISBN 0 521 63019 3 hardback
ISBN 0 521 63976 x paperback

To Libby, John, and Elizabeth

Contents

Cambridge University Press
0521630193 - General Relativity: A Geometric Approach
Malcolm Ludvigsen
Frontmatter
More information

Preface

A tribe living near the North Pole might well consider the direction defined by the North Star to be particularly sacred. It has the nice geometrical property of being perpendicular to the snow, it forms the axis of rotation for all the other stars on the celestial sphere, and it coincides with the direction in which snowballs fall. However, as we all know, this is just because the North Pole is a very special place. At all other points on the surface of the earth this direction is still special – it still forms the axis of the celestial sphere – but not that special. To the man in the moon it is not special at all.

Man's concept of *space* and *time*, and, more recently, *spacetime*, has gone through a similar process. We no longer consider the direction "up" to be special on a worldwide scale – though it is, of course, very special locally – and we no longer consider the earth to be at the center of the universe. We don't even consider the formation of the earth or even its eventual demise to be particularly special events on a cosmological scale. If we consider nonterrestrial objects, we no longer have the comfortable notion of being in the state of absolute rest (relative to what?), and, as we shall see, even the notion of straight-line, or rectilinear, motion ceases to make sense in the presence of strong gravitational fields.

All notions, theories, and ideas in physics have a certain domain of validity. The notion of absolute rest and the corresponding notion of absolute space are a case in point. If we restrict our attention to observations and experiments performed in terrestrial laboratories, then this is a perfectly meaningful and useful notion: a particle is in a state of absolute rest if it doesn't move with respect to the laboratory. In fact, that is implicitly assumed in much of elementary physics and much of quantum mechanics. However, it ceases to be meaningful if we move further afield. How, for example, do we define a state of absolute rest in outer space? Relative to the earth? Relative to the sun? The center of the galaxy, perhaps? Like the fanciful tribe's sacred direction, it must be given up (unless, of course, there does, in fact, exist a preferred, and physically detectable, state of absolute rest), and if we are to gain a clear, uncluttered view of the workings of nature, it should not enter in any way into our description of the laws of physics – it should be set aside along with the angels who once were needed to guide the planets round their orbits.

The setting aside of long-cherished but obsolete physical theories and notions is not always as easy as it might sound. It involves a new, less parochial, and less cozy way of looking at the world and, sometimes, new and unfamiliar mathematical structures. For example, after Galileo first cast doubt on the idea, it took physicists over 300 years to finally abandon the notion of absolute rest.

One of the purposes of this book is to describe, in as simple a way as possible, our present assumptions about the nature of space, time, and spacetime. I shall attempt to describe how these assumptions arise, their domain of validity, and how they can be expressed mathematically. I shall avoid speculative assumptions and theories (I shall not even mention string theories, and have very little to say about inflation) and concentrate on the bedrock of well-established theories.

Another purpose of this book is to attempt to express the laws of physics – at least those relating to spacetime – in as simple and uncluttered a form as possible, and in a form that does not rely on obsolete or (physically) meaningless notions. For example, if we agree that physical space contains no preferred point – an apparently valid assumption as far as the fundamental laws of physics are concerned – then, according to this point of view, physical space should be modeled on some sort of mathematical space containing no special point, for example, an affine space rather than a vector space. (A vector space contains a special point, namely the null vector.) In other words, I shall attempt to expel all – or, at least some – angels from the description of spacetime.

It is no accident that I use the word “spacetime” rather than “space and time” or “space-time.” It expresses the fact that, at least as far as the fundamental laws of physics are concerned, space and time form one indivisible entity. The apparent clear-cut distinction between space and time that we make in our daily lives is simply a local prejudice, not unlike the sacred direction. If, instead of going around on bicycles with a maximum speed of 10 miles per hour, we went around in spaceships with a (relative) speed of, say, 185,999 miles per second, then the distinction between space and time would be considerably less clear-cut.