

Contents

<i>Preface</i>	<i>page xv</i>
1 Background	1
1.1 Equations for fluid motion	3
1.2 Boundary conditions	9
1.3 Conservation relations	10
1.3.1 Total head and energy density conservation	11
1.3.2 Potential vorticity conservation and flux	11
1.3.3 Integral relations	12
1.4 Terminology	14
2 The flow of a homogeneous layer with a free surface	17
2.1 Basic equations	17
2.2 Flows with small obstacle height	18
2.2.1 Linear hydrostatic flow	19
2.2.2 Linear non-hydrostatic flow	26
2.3 One-dimensional non-linear hydrostatic flow	33
2.3.1 Hydraulic jumps	35
2.3.2 Flow solutions with topography	38
2.3.3 Flow through variable cross-sections and lateral contractions	44
2.3.4 Downslope flows with frictional drag	47
2.4 Non-linear waves and the <i>QRS</i> framework	55
2.5 Application to hydraulic jumps and undular bores	66
2.6 Flow over topography with non-linearity and dispersion	71
2.7 Non-linear flow past three-dimensional obstacles	77
2.7.1 Two-dimensional hydraulic jumps	79
2.7.2 Hydrostatic flow past three-dimensional obstacles	81
2.7.3 Supercritical hydrostatic flow past a varying sidewall	88

x	<i>Contents</i>	
	2.7.4 Non-hydrostatic effects and sidewalls	91
3	Two-layer flows	93
3.1	Basic equations	93
3.2	Linear waves	95
3.3	Equations for one-dimensional non-linear hydrostatic flow	96
3.4	Gravity currents	101
3.5	Two-layer hydraulic jumps	104
3.6	Hydrostatic flow over topography	110
3.7	Non-linear waves and internal bores	122
3.8	Topographic forcing with non-linearity and dispersion	129
3.9	Downstream effects	131
3.10	Two-layer flow through variable cross-sections and lateral contractions	133
	3.10.1 Two-layer flow through a channel of variable breadth	137
	3.10.2 Non-linearity with dispersion in contractions	146
3.11	Exchange flows	146
	3.11.1 Two-layer exchange flow in a uniform channel over topography	147
	3.11.2 Exchange flow through contractions	147
	3.11.3 Exchange flows through doorways and windows	162
4	Waves in stratified fluids	164
4.1	Waves in multi-layered models	164
	4.1.1 Layers with uniform density and velocity	165
	4.1.2 Layers with uniform density and vorticity	169
4.2	Continuously stratified fluids – equations	171
4.3	Stability	174
4.4	Waves in finite-depth systems	175
4.5	Waves in infinitely deep stratified fluids	181
4.6	Trapped and leaky modes	187
4.7	The effects of molecular viscosity and diffusion on internal waves	191
4.8	The process and products of the instability of shear flows	192
	4.8.1 The shear layer in homogeneous fluid	192
	4.8.2 Holmboe instability	194
	4.8.3 Disturbances in a radiating system	195
	4.8.4 Over-reflection	200

<i>Contents</i>		xi
4.9	Energy and momentum transport in a non-uniformly moving fluid	201
4.10	The “slowly varying” or WKB approximation	206
4.11	Critical layers	209
4.12	Wave-overturning and saturation	218
4.13	Wave propagation in three dimensions	219
5	Stratified flow over two-dimensional obstacles	224
5.1	Observations of flows of infinite depth	227
5.2	Infinite-depth flows: theory for small Nh/U	239
5.2.1	Small-amplitude topography with the lower boundary as a streamline	239
5.2.2	Finite topography with weak stratification, with the lower boundary as a streamline	246
5.2.3	The effects of frictional drag and lee-side separation: the obstacle as a momentum source	246
5.3	Infinite-depth flows: finite-amplitude topography and “Long’s model”	251
5.4	Infinite-depth flows with $Nh/U > (Nh/U)_c$: numerical studies	259
5.5	Linear theory for small Nh_m/U – finite depth	262
5.5.1	Small-amplitude topography with the lower boundary as a streamline	262
5.5.2	The momentum-source model	267
5.6	Comparison between linear theory, and observations and numerical results for finite depth and small Nh/U	268
5.7	Long’s-model solutions for finite depth	277
5.7.1	Rigid upper boundary	278
5.7.2	Pliant upper boundary	280
5.7.3	Upper surface with an infinitely deep stratified upper layer	284
5.8	Models with non-linearity and dispersion	285
5.9	Non-linear hydraulic flow theory for finite depth	289
5.9.1	Equations for stratified flow hydraulics	289
5.9.2	Hydraulic jumps	292
5.9.3	A procedure for obtaining steady hydraulic flow states over topography of finite height	294
5.10	Applications of the hydraulic theory	302
5.10.1	Three equal layers	302

xii	<i>Contents</i>	
	5.10.2 Three layers with a thinner upper layer	306
	5.10.3 Four equal layers	307
	5.10.4 Many equal layers, with a rigid upper boundary	307
	5.10.5 Many equal layers, with a pliant upper boundary	313
	5.11 Application of the hydraulic model to infinite-depth flows	316
	5.12 Observations and numerical results for finite Nh/U	318
	5.13 Details of the dynamics of downslope windstorms	329
	5.14 Flow across valleys	334
	5.14.1 Long's-model solutions over valleys	334
	5.14.2 Flow regimes for initially uniform U and N profiles	336
6	Stratified flow past three-dimensional topography	344
	6.1 Linear theory for small-amplitude topography, with the lower boundary as a stream surface	345
	6.1.1 Flow over periodic topography	346
	6.1.2 General solution	348
	6.1.3 Flow over short obstacles – stationary phase approximation for waves in the far-field	349
	6.1.4 Hydrostatic flow over long obstacles	356
	6.1.5 The hydrostatic flow near the ground	361
	6.1.6 Wave drag	365
	6.2 Linear theory for trapped lee waves	367
	6.3 Atmospheric lee waves	372
	6.4 Limitations and extensions of linear theory	379
	6.4.1 Flow in isosteric coordinates	380
	6.4.2 Linear hydrostatic flow in isosteric coordinates	382
	6.4.3 Numerical computations of hydrostatic nearly-linear behaviour	383
	6.5 The topology of the flow field on the surface of an obstacle	385
	6.6 Observations of the flow past three-dimensional obstacles	392
	6.6.1 More theoretical preliminaries	392
	6.6.2 Flow of homogeneous fluid ($N = 0$)	395
	6.6.3 Flow at finite Nh/U past obstacles with circular horizontal cross-section	399
	6.6.4 Flow past a sphere at finite Nh/U	408
	6.6.5 Flow at finite Nh/U past elongated obstacles	413

<i>Contents</i>	xiii
6.6.6 Flow at finite Nh/U past two-dimensional barriers with gaps	421
6.7 Flow properties for finite Nh/U – theoretical aspects	427
6.7.1 Perturbation solution for Nh/U large	427
6.7.2 The momentum-source model	432
6.7.3 Numerical studies	435
6.8 Some atmospheric examples	438
6.8.1 The island of Hawaii	438
6.8.2 The Olympic mountains	441
6.8.3 Tasmania	442
6.9 Flow past complex terrain	443
7 Applications to practical modelling of flow over complex terrain	448
7.1 Laboratory modelling	448
7.2 Parametrisation of sub-grid-scale orography in large-scale numerical models	451
7.2.1 Representation of the topography	454
7.2.2 $2N\mu/U < 1$ – the drag on the surface	456
7.2.3 The effect of atmospheric structure on the vertical distribution of gravity wave drag	458
7.2.4 $2N\mu/U > 1$ – the drag on the atmosphere	459
Appendix	463
<i>References and author index</i>	465
<i>Subject index</i>	479