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With an emphasis on both theory and experiment, this text describes the behaviour of homogeneous and density-stratified fluids over and around topography. In examining the similarities between the flow of a river over a barrier or weir and the flow of the atmosphere over a mountain range, this book presents a comprehensive synthesis of this topic in terms suitable for scientists, engineers, teachers and students of fluid dynamics. Using the appropriate mathematics, experiments, and illustrations, the text describes the properties of stratified flows beginning with the simplest situations, such as the flow of a homogeneous layer with a free surface – the prototype system for conventional hydraulics, and proceeding to progressively more complex ones, such as the flow of stratified fluid over two- and three-dimensional topography. The book concludes with a discussion of how applications of the properties and principles of these diverse phenomena may be modelled in practical terms.

With this book, Professor Peter G. Baines makes a notable contribution to the fields of fluid mechanics and geophysical fluid dynamics. The text will be a great asset to graduate and advanced undergraduate students as well as research professionals.

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***TOPOGRAPHIC EFFECTS IN  
STRATIFIED FLOWS***

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UNIVERSITY PRESS**

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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge CB2 1RP

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, United Kingdom  
40 West 20th Street, New York, NY 10011-4211, USA  
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

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First published 1995  
First paperback edition 1998

*Library of Congress Cataloging-in-Publication Data is available.*

*A catalog record for this book is available from the British Library.*

ISBN 0-521-43501-3 hardback  
ISBN 0-521-62923-3 paperback

Transferred to digital printing 2004

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*To Adrian Gill*

## Contents

<i>Preface</i>	<i>page xv</i>
<b>1 Background</b>	<b>1</b>
1.1 Equations for fluid motion	3
1.2 Boundary conditions	9
1.3 Conservation relations	10
1.3.1 Total head and energy density conservation	11
1.3.2 Potential vorticity conservation and flux	11
1.3.3 Integral relations	12
1.4 Terminology	14
<b>2 The flow of a homogeneous layer with a free surface</b>	<b>17</b>
2.1 Basic equations	17
2.2 Flows with small obstacle height	18
2.2.1 Linear hydrostatic flow	19
2.2.2 Linear non-hydrostatic flow	26
2.3 One-dimensional non-linear hydrostatic flow	33
2.3.1 Hydraulic jumps	35
2.3.2 Flow solutions with topography	38
2.3.3 Flow through variable cross-sections and lateral contractions	44
2.3.4 Downslope flows with frictional drag	47
2.4 Non-linear waves and the <i>QRS</i> framework	55
2.5 Application to hydraulic jumps and undular bores	66
2.6 Flow over topography with non-linearity and dispersion	71
2.7 Non-linear flow past three-dimensional obstacles	77
2.7.1 Two-dimensional hydraulic jumps	79
2.7.2 Hydrostatic flow past three-dimensional obstacles	81
2.7.3 Supercritical hydrostatic flow past a varying sidewall	88

x	<i>Contents</i>	
	2.7.4 Non-hydrostatic effects and sidewalls	91
<b>3</b>	<b>Two-layer flows</b>	93
3.1	Basic equations	93
3.2	Linear waves	95
3.3	Equations for one-dimensional non-linear hydrostatic flow	96
3.4	Gravity currents	101
3.5	Two-layer hydraulic jumps	104
3.6	Hydrostatic flow over topography	110
3.7	Non-linear waves and internal bores	122
3.8	Topographic forcing with non-linearity and dispersion	129
3.9	Downstream effects	131
3.10	Two-layer flow through variable cross-sections and lateral contractions	133
	3.10.1 Two-layer flow through a channel of variable breadth	137
	3.10.2 Non-linearity with dispersion in contractions	146
3.11	Exchange flows	146
	3.11.1 Two-layer exchange flow in a uniform channel over topography	147
	3.11.2 Exchange flow through contractions	147
	3.11.3 Exchange flows through doorways and windows	162
<b>4</b>	<b>Waves in stratified fluids</b>	164
4.1	Waves in multi-layered models	164
	4.1.1 Layers with uniform density and velocity	165
	4.1.2 Layers with uniform density and vorticity	169
4.2	Continuously stratified fluids – equations	171
4.3	Stability	174
4.4	Waves in finite-depth systems	175
4.5	Waves in infinitely deep stratified fluids	181
4.6	Trapped and leaky modes	187
4.7	The effects of molecular viscosity and diffusion on internal waves	191
4.8	The process and products of the instability of shear flows	192
	4.8.1 The shear layer in homogeneous fluid	192
	4.8.2 Holmboe instability	194
	4.8.3 Disturbances in a radiating system	195
	4.8.4 Over-reflection	200

<i>Contents</i>		xi
4.9	Energy and momentum transport in a non-uniformly moving fluid	201
4.10	The “slowly varying” or WKB approximation	206
4.11	Critical layers	209
4.12	Wave-overturning and saturation	218
4.13	Wave propagation in three dimensions	219
<b>5</b>	<b>Stratified flow over two-dimensional obstacles</b>	<b>224</b>
5.1	Observations of flows of infinite depth	227
5.2	Infinite-depth flows: theory for small $Nh/U$	239
5.2.1	Small-amplitude topography with the lower boundary as a streamline	239
5.2.2	Finite topography with weak stratification, with the lower boundary as a streamline	246
5.2.3	The effects of frictional drag and lee-side separation: the obstacle as a momentum source	246
5.3	Infinite-depth flows: finite-amplitude topography and “Long’s model”	251
5.4	Infinite-depth flows with $Nh/U > (Nh/U)_c$ : numerical studies	259
5.5	Linear theory for small $Nh_m/U$ – finite depth	262
5.5.1	Small-amplitude topography with the lower boundary as a streamline	262
5.5.2	The momentum-source model	267
5.6	Comparison between linear theory, and observations and numerical results for finite depth and small $Nh/U$	268
5.7	Long’s-model solutions for finite depth	277
5.7.1	Rigid upper boundary	278
5.7.2	Pliant upper boundary	280
5.7.3	Upper surface with an infinitely deep stratified upper layer	284
5.8	Models with non-linearity and dispersion	285
5.9	Non-linear hydraulic flow theory for finite depth	289
5.9.1	Equations for stratified flow hydraulics	289
5.9.2	Hydraulic jumps	292
5.9.3	A procedure for obtaining steady hydraulic flow states over topography of finite height	294
5.10	Applications of the hydraulic theory	302
5.10.1	Three equal layers	302



5.10.2	Three layers with a thinner upper layer	306
5.10.3	Four equal layers	307
5.10.4	Many equal layers, with a rigid upper boundary	307
5.10.5	Many equal layers, with a pliant upper boundary	313
5.11	Application of the hydraulic model to infinite-depth flows	316
5.12	Observations and numerical results for finite $Nh/U$	318
5.13	Details of the dynamics of downslope windstorms	329
5.14	Flow across valleys	334
5.14.1	Long's-model solutions over valleys	334
5.14.2	Flow regimes for initially uniform $U$ and $N$ profiles	336
<b>6</b>	<b>Stratified flow past three-dimensional topography</b>	<b>344</b>
6.1	Linear theory for small-amplitude topography, with the lower boundary as a stream surface	345
6.1.1	Flow over periodic topography	346
6.1.2	General solution	348
6.1.3	Flow over short obstacles – stationary phase approximation for waves in the far-field	349
6.1.4	Hydrostatic flow over long obstacles	356
6.1.5	The hydrostatic flow near the ground	361
6.1.6	Wave drag	365
6.2	Linear theory for trapped lee waves	367
6.3	Atmospheric lee waves	372
6.4	Limitations and extensions of linear theory	379
6.4.1	Flow in isosteric coordinates	380
6.4.2	Linear hydrostatic flow in isosteric coordinates	382
6.4.3	Numerical computations of hydrostatic nearly-linear behaviour	383
6.5	The topology of the flow field on the surface of an obstacle	385
6.6	Observations of the flow past three-dimensional obstacles	392
6.6.1	More theoretical preliminaries	392
6.6.2	Flow of homogeneous fluid ( $N = 0$ )	395
6.6.3	Flow at finite $Nh/U$ past obstacles with circular horizontal cross-section	399
6.6.4	Flow past a sphere at finite $Nh/U$	408
6.6.5	Flow at finite $Nh/U$ past elongated obstacles	413

<i>Contents</i>		xiii
6.6.6	Flow at finite $Nh/U$ past two-dimensional barriers with gaps	421
6.7	Flow properties for finite $Nh/U$ – theoretical aspects	427
6.7.1	Perturbation solution for $Nh/U$ large	427
6.7.2	The momentum-source model	432
6.7.3	Numerical studies	435
6.8	Some atmospheric examples	438
6.8.1	The island of Hawaii	438
6.8.2	The Olympic mountains	441
6.8.3	Tasmania	442
6.9	Flow past complex terrain	443
<b>7</b>	<b>Applications to practical modelling of flow over complex terrain</b>	<b>448</b>
7.1	Laboratory modelling	448
7.2	Parametrisation of sub-grid-scale orography in large-scale numerical models	451
7.2.1	Representation of the topography	454
7.2.2	$2N\mu/U < 1$ – the drag on the surface	456
7.2.3	The effect of atmospheric structure on the vertical distribution of gravity wave drag	458
7.2.4	$2N\mu/U > 1$ – the drag on the atmosphere	459
	<b>Appendix</b>	<b>463</b>
	<i>References and author index</i>	465
	<i>Subject index</i>	479

## Preface

This project was conceived about ten years ago, but the incentive to pursue and complete it was hampered until recently because of several fundamental unresolved questions about the nature of stratified flow around topography. Within the last few years it has become possible to answer these questions, as a result of the efforts of several people, and the answers are embodied in the synthesis presented here.

Who will benefit from purchasing, reading or thumbing through this book? It is primarily addressed to fluid dynamicists, meteorologists, oceanographers, engineers, physicists and mathematicians who wish to learn more about the dynamics of stratified fluids. Some background in fluid dynamics is probably necessary, but the subject is treated from first principles and is developed from simple situations toward more complex ones. Overall, the order of presentation is based on logic rather than the historical development. There is balance between theory and experiment, where the comparison is made whenever possible, and a consistent attempt has been made to provide a physical understanding of the phenomena involved.

I have gone to some length to make the material easily assimilable, as the number of figures testifies. As I see it, a book such as this is the next step in the scientific process of the documentation of a subject, following the initial “source” material in journals. It is a documented attempt to digest such material, and should therefore be easier to read. However, much of the material presented here is new, as part of the process of filling gaps and providing a (more) complete picture. A number of new experiments have been carried out at Aspendale specifically for this volume.

I have attempted to give an adequate list of references so that readers can delve deeper into the subject, but it is not exhaustive, and

some relevant work may have been omitted. I apologise in advance to any colleagues to whom due reference has not been given.

In its present form, this book has been made possible by the dedicated and professional efforts of several people at Aspendale: most notably David Murray, who has played a major part in most of my experimental studies over the past ten years; David Whillas, whose talents are evident in several photographs; and Sean Higgins, who has skillfully adapted and created many of the line drawings. Thanks are also due to others who have provided continual background support.

I am grateful to several colleagues who have contributed photographs or figures, and to various copyright holders for permission to use some figures, and these are acknowledged in the captions. I am also grateful to others who have taken the time to read and comment on drafts of chapters in varying degrees of imperfection, and specifically these include Jim Rottman in particular, and Ian Castro, Terry Clark, Jack Katzfey, Peter Killworth, Greg Lawrence, Mike Sewell, Bill Snyder, Larry Armi and Sharan Majumdar. Thanks are also due to numerous colleagues for informative discussion on the material of this book over many years, to George Batchelor for his advice and support, and to Alan Harvey, Brian Watts and the staff of Cambridge University Press for their cooperation and attention to detail.

Aspendale  
September, 1993 & December 1994

*Peter G. Baines*