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*DYNAMIC FRACTURE MECHANICS*

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L. B. Freund  
Frontmatter  
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# *Dynamic Fracture Mechanics*

*L. B. FREUND*  
*Brown University*



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## PREFACE

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This book is an outgrowth of my involvement in the field of dynamic fracture mechanics over a period of nearly twenty years. This sub-branch of fracture mechanics has been wonderfully rich in scope and diversity, attracting the attention of both researchers and practitioners with backgrounds in the mechanics of solids, applied mathematics, structural engineering, materials science, and earth science. A wide range of analytical, experimental, and computational methods have been brought to bear on the area. Overall, the field of dynamic fracture is highly interdisciplinary, it provides a wealth of challenging fundamental issues for study, and new results have the potential for immediate practical application. In my view, this combination of characteristics accounts for its continued vitality.

I have written this book in an effort to summarize the current state of the mechanics of dynamic fracture. The emphasis is on fundamental concepts, the development of mathematical models of phenomena which are dominated by mechanical features, and the analysis of these models. Mathematical problems which are representative of the problem classes that comprise the area are stated formally, and they are also described in common language in an effort to make their features clear. These problems are solved using mathematical methods that are developed to the degree required to make the presentation more or less self-contained. Experimental and computational approaches have been of central importance in this field, and relevant results are cited in the course of discussion. The extraordinary contributions of the few individuals who pioneered the area of dynamic fracture mechanics occupy prominent positions in this discussion. One hope in preparing this book is that people with new perspectives will be attracted to the field, which continues to provide

fascinating and technically important challenges. Perhaps the book can serve as a guide to further development of the area.

The reader is assumed to be familiar with concepts of continuum mechanics and methods of applied mathematics to the level normally provided through the first year of study in a graduate program in solid mechanics in the United States. A brief summary of relevant results is included in the first chapter in order to establish notation and to provide a common source for reference in later chapters. Some background in equilibrium fracture mechanics would be helpful, of course, but none is presumed. In terms of graduate instruction in fracture mechanics, the book could serve as a text for a course devoted to dynamic fracture mechanics or, for a more general course, as a supplement to other books which provide broader coverage of the whole of fracture mechanics. The overall organization of the book is evident from the chapter titles. The brief overview included in Chapter 1 can serve as a guide to those readers interested in using the book as a source of reference for specific results.

The bibliography is an important part of the book. In view of contemporary publication practices, the compilation of an all-inclusive bibliography in any technical area is an impossible task. Nonetheless, the bibliography is intended to be comprehensive in the sense that it includes entries which describe research results on essentially all aspects of dynamic fracture. With only a few exceptions, the entries are either articles published in the open literature or relevant textbooks and monographs. Thus, most of the references should be available in a reasonably complete technical library. All references cited in the text are included in the bibliography, and many additional references are included as well. Some judgment was required in the selection of references for citation in the text, and I have done my best to accurately identify sources for key steps in the evolution of ideas.

I am indebted to a number of colleagues who read drafts of various chapters of the book. Those who offered suggestions and encouragement in this way are John Hutchinson, Fred Nilsson, Ares Rosakis, and John Willis. A special thanks goes to Jim Rice, Editor of this series, who generously read a draft of the entire manuscript. It has been among my greatest fortunes to be a member of the Solid Mechanics Group at Brown University, which has provided an intellectually stimulating and most congenial environment over the years. I am especially grateful to my colleagues Rod Clifton, Alan Needleman, Michael Ortiz, Fong Shih, and Jerry Weiner for their willingness to

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read and discuss some of the material that is included here. My own views on the mechanics of dynamic fracture and its fundamental precepts have been formed over a long period of time through interactions with people far too numerous to mention individually. This group includes the many colleagues and students with whom I have collaborated and written joint papers; they are identified in the bibliography. It also includes those who, through questions and discussions, showed me where my own understanding of certain points had been incomplete.

I also thank Peter-John Leone, Earth Sciences Editor, Rhona Johnson, Manuscript Development Editor, and Louise Calabro Gruendel, Production Editor, of Cambridge University Press in New York for the efficient way in which they have managed the preparation of the book and for their sensitivity in dealing with my concerns on the matter. It has been a pleasure to be involved in long-term programs of research on fracture mechanics funded by the Office of Naval Research and by the National Science Foundation. These programs, and the collaborations that they have fostered, have been invaluable.

Finally, I thank my wife Colleen and our sons, Jon, Jeff, and Steve, who enthusiastically adopted the mission of writing this book as their own. Their interest and unwavering devotion have lightened the task immensely.

*L. B. Freund*

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## LIST OF SYMBOLS

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Mathematical symbols and functions are defined the first time that they are used in the book. Brief definitions of the most frequently used symbols and functions are listed below. Some symbols necessarily have different definitions in different sections. In such cases, definitions are stated locally and are used consistently within sections.

$a$	Inverse dilatational wave speed $c_d^{-1}$ ; half length of a Griffith crack
$A_I(v)$	Universal function of crack tip speed for mode I deformation (similar for modes II and III)
$b$	Inverse shear wave speed $c_s^{-1}$ ; a material parameter
$c$	Inverse Rayleigh wave speed $c_R^{-1}$ ; an elastic wave speed
$c_d$	Elastic dilatational wave speed
$c_o$	Speed of longitudinal waves in an elastic bar
$c_R$	Elastic Rayleigh surface wave speed
$c_s$	Elastic shear wave speed
$C_I$	Dimensionless factor for the mode I asymptotic stress field (similar for modes II and III)
$C_{ijkl}$	Components of the elastic stiffness tensor
$d$	Inverse crack tip speed $v^{-1}$
$d_{ij}$	Components of the symmetric part of the velocity gradient tensor
$D(v)$	The quantity $4\alpha_d\alpha_s - (1 + \alpha_s^2)^2$ ; a function of crack tip speed
$E$	Young's elastic modulus
$E_R$	Far-field radiated energy
$F(\Gamma)$	Energy flux through contour $\Gamma$
$G$	Energy release rate; dynamic energy release rate

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$G_a$	Crack arrest fracture energy
$G_c$	Critical value of energy release rate
$h$	Thickness of a beam; width of a strip
$h_i(x, t)$	Components of the dynamic weight function
$H(t)$	Unit step function
$J_N(\Gamma, s)$	Path independent integral of Laplace transformed fields
$k(v)$	Universal function of crack tip speed for elastic crack growth in mode I
$k_{II}(v)$	Universal function of crack tip speed for elastic crack growth in mode II (similar for mode III)
$K_I$	Elastic stress intensity factor for mode I (similar for modes II and III)
$K_{Ia}$	Value of stress intensity factor at crack arrest; crack arrest toughness (similar for modes II and III)
$K_{Iappl}$	Remotely applied stress intensity factor
$K_{Ic}$	Value of stress intensity factor at fracture initiation; fracture toughness (similar for modes II and III)
$K_{Id}$	Value of stress intensity factor during crack growth; dynamic fracture toughness (similar for modes II and III)
$l, l(t)$	Crack length; amount of crack growth
$\dot{l}$	Instantaneous crack tip speed
$l^\pm$	Limit as position $x = l$ is approached through values of $x$ greater than $l$ (+) or less than $l$ (-)
$l_c$	Critical crack length
$l_0$	Initial crack length
$m$	Normalized crack tip speed $v/c_s$ or $v/c_R$
$M_{ijkl}$	Components of the elastic compliance tensor
$n$	Crack tip bluntness parameter; a material parameter
$n_i$	Components of unit vector; normal to surface or curve
$o(f(x))$	Asymptotically dominated by $f$ as $x \rightarrow$ a limit point
$O(f(x))$	Asymptotically proportional to $f$ as $x \rightarrow$ a limit point
$p(x)$	Pressure distribution
$p^*$	Magnitude of a concentrated normal force
$P(\zeta)$	Amplitude of $\Phi$
$P_c$	Dimensionless combination of material parameters for a strain-rate-dependent elastic-plastic solid
$q^*$	Magnitude of a concentrated shear force
$Q(\zeta)$	Amplitude of $\Psi$

$r$	Polar coordinate
$r_d \exp(i\theta_d)$	Polar form of the complex variable $x + i\alpha_d y$
$r_p$	Plastic zone size
$r_s \exp(i\theta_s)$	Polar form of the complex variable $x + i\alpha_s y$
$R$	Region in space; region in a plane
$R(\zeta)$	Rayleigh wave function
$s$	Laplace transform parameter; a real variable
$s_{ij}$	Components of the deviatoric stress tensor
$S_{\pm}(\zeta)$	Factors of the Rayleigh wave function that are nonzero and analytic in overlapping half planes
$t$	Time coordinate
$T$	Kinetic energy density
$T_{tot}$	Total kinetic energy of a body
$\mathbf{T}, \mathcal{T}_i$	Traction vector; components of traction vector
$u_i$	Components of particle displacement vector ( $i = 1, 2, 3$ or $i = x, y, z$ )
$\dot{u}_i$	Components of particle velocity vector
$\ddot{u}_i$	Components of particle acceleration vector
$u_y^L(x, t)$	Normal surface displacement for Lamb's problem
$u_-(x, t)$	Displacement distribution for $x < 0$
$U$	Stress work density; elastic strain energy density
$U_{tot}$	Total stress work; total strain energy
$v$	Crack tip speed
$w$	Displacement for antiplane shear deformation
$x_i$	Rectangular coordinates ( $i = 1, 2, 3$ )
$x, y, z$	Rectangular coordinates
$\alpha(\zeta)$	The function $(a^2 - \zeta^2)^{1/2}$
$\alpha_d, \alpha_s$	The quantities $\sqrt{1 - v^2/c_d^2}$ and $\sqrt{1 - v^2/c_s^2}$
$\beta(\zeta)$	The function $(b^2 - \zeta^2)^{1/2}$
$\gamma$	A material parameter
$\dot{\gamma}_0$	Viscosity parameter of a rate dependent plastic material
$\Gamma$	Crack tip contour; specific fracture energy
$\Gamma(\cdot)$	Gamma (factorial) function
$\Gamma_c, \Gamma_m, \Gamma_0$	Constant values of specific fracture energy
$\delta_t$	Crack tip opening displacement
$\delta(t)$	Dirac delta function
$\Delta$	Amplitude of an elastic dislocation
$\epsilon$	A small real parameter
$\epsilon_{ij}$	Components of the small strain tensor

*Symbols*

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$\epsilon_{ij}^e$	Components of the elastic strain tensor
$\epsilon_{ij}^p$	Components of the plastic strain tensor
$\zeta$	Complex variable; Laplace transform parameter
$\eta$	Rectangular coordinate
$\theta$	Polar coordinate
$\kappa$	Parameter determined by the behavior of the Rayleigh wave function as $ \zeta  \rightarrow \infty$
$\lambda$	Lamé elastic constant
$\Lambda$	Length of cohesive zone
$\mu$	Lamé elastic constant; elastic shear modulus
$\nu$	Poisson's ratio
$\nu, \nu_i$	Unit vector normal to a surface or curve
$\xi$	Rectangular coordinate
$\rho$	Material mass density
$\sigma$	Mean stress; effective stress
$\sigma(x)$	Normal traction within a cohesive zone
$\sigma_{ij}$	Components of the stress tensor
$\sigma_\infty$	Amplitude of remotely applied tension
$\sigma_0$	Tensile flow stress of an ideally plastic material
$\sigma_+(x, t)$	Normal traction distribution for $x > 0$
$\sigma^*$	Magnitude of applied normal traction
$\Sigma_{ij}^I$	Angular variation of asymptotic crack tip stress field for mode I (similar for modes II and III)
$\tau_\infty$	Amplitude of remotely applied shear traction
$\tau_0$	Shear flow stress of an ideally plastic material
$\tau_+(x, t)$	Shear traction distribution on $x > 0$
$\tau^*$	Magnitude of applied shear traction
$\phi$	Lamé scalar displacement potential function
$\Phi$	Double Laplace transform of $\phi$
$\psi, \psi_i$	Lamé vector displacement potential function
$\psi$	Magnitude of $\psi$ for plane deformation; local shear angle in plastically deforming region
$\Psi$	Double Laplace transform of $\psi$
$\Omega$	Potential energy
$\Omega_S$	Potential energy increase due to creation of free surface
$(cc.ss.nn)$	Equation number $nn$ in Section $ss$ of Chapter $cc$
$(c.s.n)_m$	The $m$ th equation in a group of equations identified by the single number $(c.s.n)$