1

BACKGROUND AND OVERVIEW

1.1 Introduction

The field of fracture mechanics is concerned with the quantitative description of the mechanical state of a deformable body containing a crack or cracks, with a view toward characterizing and measuring the resistance of materials to crack growth. The process of describing the mechanical state of a particular system is tantamount to devising a mathematical model of it, and then drawing inferences from the model by applying methods of mathematical or numerical analysis. The mathematical model typically consists of an idealized description of the geometrical configuration of the deformable body, an empirical relationship between internal stress and deformation, and the pertinent balance laws of physics dealing with mechanical quantities. For a given physical system, modeling can usually be done at different levels of sophistication and detail. For example, a particular material may be idealized as being elastic for some purposes but elastic-plastic for other purposes, or a particular body may be idealized as a one-dimensional structure in one case but as a three-dimensional structure in another case. It should be noted that the results of most significance for the field have not always been derived from the most sophisticated and detailed models.

A question of central importance in the development of a fracture mechanics theory is the following. Is there any particular feature of the mechanical state of a cracked solid that can be interpreted as a "driving force" acting on the crack, that is, an effect that is correlated with a tendency for the crack to extend? A viewpoint that underlies this important concept is that of the crack as an entity which itself behaves according to a "law" of mechanics expressed in terms of a relationship between driving force and motion. The modeling phase provides the language for considering the strength of real materials, and it concludes with hypotheses on the behavior of materials. It is only through observation of the fracturing of materials that the hypotheses can be verified or refuted. This synergistic process has led to standard methods of practice whereby fracture mechanics is used routinely in materials selection and engineering design.

1.1.1 Inertial effects in fracture mechanics

Dynamic fracture mechanics is the subfield of fracture mechanics concerned with fracture phenomena for which the role of material inertia becomes significant. Phenomena for which strain rate dependent material properties have a significant effect are also typically included. Inertial effects can arise either from rapidly applied loading on a cracked solid or from rapid crack propagation. In the case of rapid loading, the influence of the loads is transferred to the crack by means of stress waves through the material. To determine whether or not a crack will advance due to the stress wave loading, it is necessary to determine the transient driving force acting on the crack. In the case of rapid crack propagation, material particles on opposite crack faces displace with respect to each other once the crack edge has passed. The inertial resistance to this motion can also influence the driving force, and it must be taken into account in a complete description of the process. There is also a connection between the details of rapid crack motion and the stress wave field radiated from a moving crack that is important in seismology, as well as in some material testing techniques.

Progress toward understanding dynamic fracture phenomena has been impeded by several complicating features. The inherent time dependence of a dynamic fracture process results in mathematical models that are more complex than the equivalent equilibrium models for the same configuration and material class. From the experimental point of view, the time dependence requires that many accurate sequential measurements of quantities of interest must be made in an extremely short time period in a way that does not interfere with the process being observed. In the case of crack growth, the regions of the boundary of the body over which certain conditions must be enforced in a mathematical model change with time. From the experimental point of view, this feature implies that the place where quantities of interest must be measured varies, usually in a nonuniform way, during the process.

The question of whether or not inertial effects are significant in any given fracture situation depends on the loading conditions, the material characteristics, and the geometrical configuration of the body. Circumstances under which they are indeed significant cannot be specified unambiguously, but some guidelines are evident. For example, suppose that there is a characteristic time associated with the applied loading, say a load maximum divided by the rate of load increase. Unless this time is large compared to the time required for a stress wave to travel at a characteristic wave speed of the material over a representative length of the body, say the crack length or the distance from the crack edge to the loaded boundary, it can be expected that inertial effects will be significant. In the case of crack growth, inertial effects will probably be important if the speed of the crack tip or edge is a significant fraction of the lowest characteristic wave speed of the material. If the boundaries of the body must be taken into account, then the potential for the body to function as a waveguide must be considered. The group velocity of guided waves is frequently much less than the characteristic bulk wave speeds of the material. For such cases, inertial effects could be important for crack speeds that are much less than the bulk wave speeds but comparable to relevant group velocities.

1.1.2 Historical origins

Empirical studies concerned with the bursting of military cannon or with the impact loading of industrial machines were carried out in the 19th century. For example, an interesting early series of observations on "new" industrial iron-based materials is reported by Kirkaldy (1863). However, no single scientific discovery or other event can be identified as the stimulus for launching dynamic fracture mechanics as an area of research. Instead, the area as it is summarized in this book is the conjunction of numerous paths of investigation that were motivated by pressing practical needs in engineering design and material selection, by scientific curiosity about earthquakes and other natural phenomena, by challenges of classes of mathematical problems, and by advances in experimental and observational techniques.

The area has been under continuous development as a subfield of the engineering science of fracture mechanics since the 1940s and as a topical area in geophysics since the late 1960s. The view in the early days of fracture mechanics was that once a crack in a structure began to grow the structure had failed (Tipper 1962). The rapid crack growth phase of failure was viewed as interesting but not particularly important. Beginning about 1970, however, the importance of understanding crack propagation and arrest was recognized in both engineering and earth sciences, and significant progress has been made since then.

A few of the early contributions to the area that provided ideas of lasting significance are cited here. In some cases, these ideas were developed in connection with fracture under equilibrium conditions, but they have been important in the area of dynamic fracture as well. Among these is the work of Griffith (1920), which is commonly acknowledged as representing the start of equilibrium fracture mechanics as a quantitative science of material behavior. In considering an ideally brittle elastic body containing a crack, he recognized that the macroscopic potential energy of the system, consisting of the internal stored elastic energy and the external potential energy of the applied loads, varied with the size of the crack. He also recognized that extension of the crack resulted in the creation of new crack surface, and he postulated that a certain amount of work per unit area of crack surface must be expended at a microscopic level to create that area. In this context, the term "microscopic" implies that this work is not included in a continuum description of the process. It is common to characterize this work per unit area by assuming that a certain force-displacement relationship governs the reversible interaction of atoms or molecules across the fracture plane. The area under the force-displacement relationship from equilibrium to full separation, averaged over the fracture surface, is assumed to represent this work of separation. This is one way to define the *surface energy* of the material. Griffith simply included this work as an additional potential energy of the system, and then invoked the equilibrium principle of minimum potential energy for conservative systems. That is, he considered the system to be in equilibrium with a particular fixed loading and a particular crack length. He postulated that the crack was at a *critical state of incipient growth* if the reduction in macroscopic potential energy associated with a small virtual crack advance from that state was equal to the microscopic

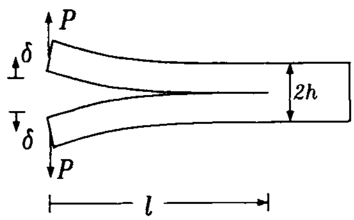

Figure 1.1. A crack of length l in a rectangular strip. Forces P act to separate the crack faces and the displacement of each load point is δ.

work of separation for the crack surface area created by the virtual crack advance. A particular attraction of Griffith's energy fracture condition is that it obviates the need to examine the actual fracture process at the crack tip in detail.

Griffith's original work was based on the plane elasticity solution for a crack of finite length in a body subjected to a uniform remote tension in a direction normal to the crack plane. The concept is illustrated here by means of a simple example based on elementary elastic beam theory. Consider the split rectangular strip shown in Figure 1.1 which is of unit thickness in the direction perpendicular to the plane of the figure. Suppose that the slit or crack of length l is opened symmetrically by imposing either forces or displacements at the ends of the arms as shown. In either case, the end force per unit thickness is denoted by P and the displacement of the load point of each arm from the undeflected equilibrium position is denoted by δ. Suppose that the isotropic material is elastic with Young's modulus E and that the deformation can be approximated by assuming that each arm deforms as a Bernoulli-Euler beam of length l cantilevered at the crack tip end. The relationship between the end force per unit thickness and the end displacement is then

$$P = \frac{Eh^3}{4l^3}\delta \tag{1.1.1}$$

and the total stored elastic energy per unit thickness in both arms is

$$\delta P = \frac{Eh^3}{4l^3}\delta^2 = \frac{4l^3}{Eh^3}P^2 \,. \tag{1.1.2}$$

The work of separation per unit area of surface created is denoted by γ.

Consider first the case of imposed end displacement δ^*. What is the critical crack size for this fixed end displacement according to the Griffith theory? In this case, there is no energy exchange between the body and its surroundings during the virtual crack extension, and the macroscopic potential energy per unit thickness Ω is the stored elastic energy $\Omega = Eh^3\delta^{*2}/4l^3$. The total work-of-fracture per unit thickness is γ times the crack length for both faces, or $\Omega_S = 2\gamma l$. The total work-of-fracture per unit thickness could equally well be written as 2γ times the amount of crack extension from some arbitrary initial crack length, a work measure that would differ from Ω_S by a constant. This constant is arbitrary, and it has no significance in considering crack advance. The total potential energy is $\Omega + \Omega_S$. According to the Griffith postulate, the critical crack length for incipient growth is determined by the condition

$$\frac{\partial}{\partial l}(\Omega + \Omega_S) = 0 \quad \Rightarrow \quad l_c = \left[\frac{3Eh^3\delta^{*2}}{8\gamma}\right]^{1/4} . \tag{1.1.3}$$

The individual potential energies vary with crack length as shown schematically in Figure 1.2. Note that

$$\frac{\partial^2}{\partial l^2}(\Omega + \Omega_S) > 0 \tag{1.1.4}$$

at the critical crack length. This means that the system is stable in the sense of mechanical equilibrium. If the crack length is at the critical value and δ^* is slowly increased to a larger value, then the crack will slowly advance under equilibrium conditions to a new length that is related to the increased δ^* through $(1.1.3)_2$. A useful interpretation of $(1.1.3)_1$ is that $-\partial\Omega/\partial l$ is the feature of the mechanical state that can be identified as a crack driving force. The driving force is work-conjugate to crack position l, and the fracture mechanics balance law governing crack behavior is $-\partial\Omega/\partial l = 2\gamma$.

Consider next the case of imposed end force P^*. What is the critical crack size for this end condition? The stored elastic energy is again given by (1.1.2). The external potential energy of the loading on both arms is $-2\delta P^* = -8l^3P^{*2}/Eh^3$, so that the macroscopic

Cambridge University Press
978-0-521-62922-5 - Dynamic Fracture Mechanics
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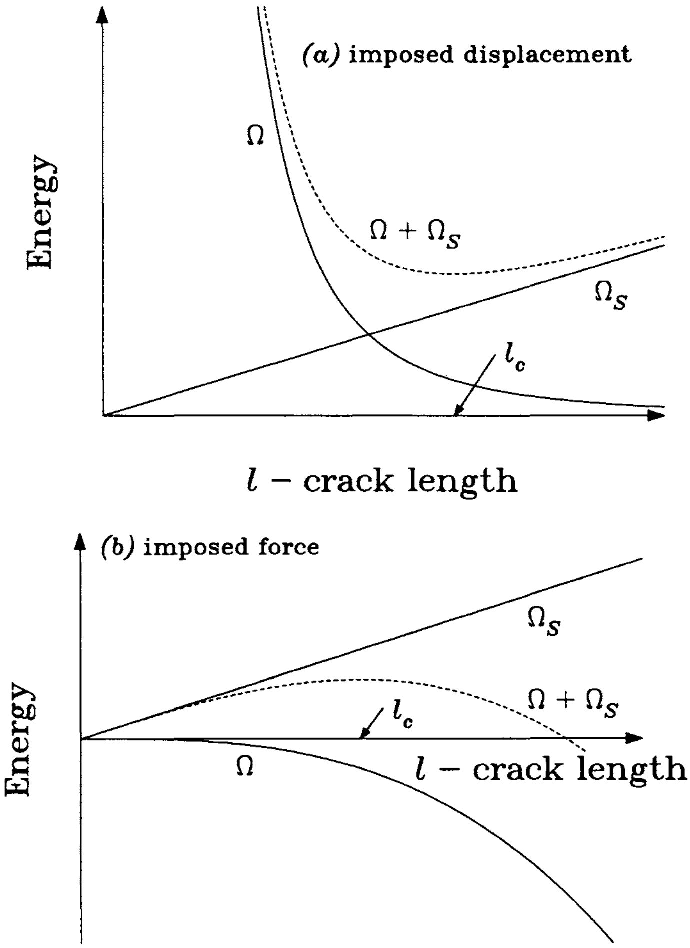

Figure 1.2. The variation of energy with crack length for the split rectangular strip in Figure 1.1 for (a) imposed displacement or (b) imposed force. The variation of the continuum potential energy plus the surface energy is shown by the dashed line.

potential energy is $\Omega = -4l^3P^{*2}/Eh^3$. Imposition of the Griffith condition in this case yields

$$\frac{\partial}{\partial l}(\Omega + \Omega_S) = 0 \quad \Rightarrow \quad l_c = \left[\frac{Eh^3\gamma}{6P^{*2}}\right]^{1/2}. \qquad (1.1.5)$$

Again, the way in which the individual potential energies vary with crack size is illustrated schematically in Figure 1.2. If δ^*, P^*, and

l_c are related as in (1.1.1) then the critical lengths in (1.1.3b) and (1.1.5b) are identical. However, there is a fundamental difference between the two cases. In the latter case,

$$\frac{\partial^2}{\partial l^2}\,(\Omega + \Omega_S) < 0 \tag{1.1.6}$$

at the critical crack length. This means that the system is unstable in an equilibrium sense. When $l = l_c$ for a given P^*, the state is an equilibrium state of incipient crack growth. However, if P^* is increased by any amount from this state, then it is no longer possible to find an equilibrium configuration. Instead, the crack begins to grow rapidly and inertial resistance to motion is called into play to ensure balance of momentum. This idea is fully developed in Chapter 7 in discussing dynamic crack growth.

A theoretical framework for including inertial effects during the rapid crack growth phase was first proposed by Mott (1948), who adopted the features of Griffith's analysis as a point of departure. He recognized that inertial resistance of the material to crack opening could become significant at high crack speeds. To estimate the crack speed for a particular loading system, he assumed that the crack growth process was steady state, that is, time independent as seen by an observer moving at speed v with the crack tip. Under these conditions, he obtained an estimate for the total kinetic energy of the system T_{tot} in the form of v^2 times a function of crack length l. He then argued that the total energy of the system $\Omega + \Omega_S + T$ is constant, so that

$$\frac{\partial}{\partial l}\,(\Omega + T_{tot} + \Omega_S) = 0\,. \tag{1.1.7}$$

This condition provides a relationship involving the loading parameters, the crack length l, and the crack speed v. In implementing this condition for the Griffith plane elasticity crack model, Mott assumed that Ω and T could be calculated on the basis of the equilibrium field.

The idea underlying (1.1.7) is correct if the terms are strictly interpreted. In fact, it is a special case for steady state crack growth of the general energy-rate balance: Rate of work of applied loads = rate of increase of stored elastic energy + rate of increase of kinetic energy − rate of energy loss at the crack tip. However, later work has shown that the assumptions made by Mott in his original analysis of the dynamic Griffith crack are not valid. Consequently, the conclusions

Cambridge University Press
978-0-521-62922-5 - Dynamic Fracture Mechanics
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inferred from this model are generally not valid. For example, this analysis has often been cited as establishing that the maximum crack speed in an ideal brittle material is some fixed fraction, typically given as about one-half, of the shear wave speed of the material. If the model is analyzed more thoroughly, it is found that this is not the case. Indeed, the result that the theoretical limiting speed of a tensile crack must be the Rayleigh wave speed, as established in Chapter 7, was anticipated by Stroh (1957) on the basis of an intuitive argument. Nonetheless, Mott's basic idea of energy balance during rapid crack growth was very important to the development of the subject. A number of energy concepts pertinent to dynamic fracture are discussed in Chapter 5, and the crack tip equation of motion of the type originally sought by Mott is presented in Chapter 7.

Another important theoretical idea for applying work and energy methods to fracture dynamics was provided by Irwin (1948). He was concerned with cleavage fracture of structural steels, a cleavage process that is invariably accompanied by some amount of plastic flow of the material adjacent to the fracture path. Irwin adopted Mott's postulate of stationary total energy, including a work of fracture, denoted above by Ω_S. However, Irwin proposed that this term Ω_S may be approximately represented as the sum of two terms, one proportional to the area of fracture surface and the other proportional to the volume of material affected by plastic deformation. Within the context of the Griffith theory outlined above, this assumption is invoked by writing $\Omega_S = 2(\gamma_s + \gamma_p)l$ when the thickness of the plastically deformed layer adjacent to the crack faces is small. Thus, γ_s represents the surface energy of the material associated with cleavage fracture and γ_p represents the plastic work dissipated in the surrounding material per unit crack surface area created. Irwin's idea therefore extended the range of applicability of the energy balance fracture theory to include materials that undergo some plastic deformation during crack growth. This particular extension of Griffith's theory was also proposed for fracture initiation and crack propagation in steels by Orowan (1955). Early experiments aimed at measuring γ_p for cleavage of metal single crystals and polycrystals were carried out by Hall (1953). He showed that $\gamma_p \gg \gamma_s$ for metals, particularly for polycrystals. Furthermore, both Hall and Irwin predicted strong connections between the fracture behavior of polycrystalline metals and their microstructures. An extensive survey of early work on the connection between metallurgical properties and brittle fracture of welded steel plates was given by Wells (1961) and Tipper (1962).

The continuum field approach to fracture of solids was launched with the introduction of the *elastic stress intensity factor*, usually denoted by K, as a crack tip field characterizing parameter by Irwin (1957). This idea provided an alternate framework for discussing the strength of cracked solids of nominally elastic material. Irwin proposed that a crack will begin to grow in a cracked body with limited plastic deformation when K is increased to a value called the *fracture toughness* of the material. The equivalence of the Irwin stress intensity factor criterion and the Griffith energy criterion for onset of growth of a tensile crack in a two-dimensional body of nominally elastic material under plane stress conditions was demonstrated by Irwin (1957, 1960), who showed that

$$-\frac{\partial \Omega}{\partial l} = \frac{K^2}{E}, \tag{1.1.8}$$

where E is Young's modulus of the material. The quantity on the left side of (1.1.8) is usually denoted by G and it is called the *energy release rate*. Much of the analysis of this book is devoted to determining the stress intensity factor or the energy release rate for various conditions. These concepts are developed more thoroughly in a way relevant to dynamic fracture mechanics in the chapters to follow.

Another idea that has been important in the evolution of fracture mechanics concerns the size scale over which different phenomena dominate a fracture process. The idea is implicit in Irwin's stress intensity factor concept and it is a central feature of the crack tip cohesive zone model introduced by Barenblatt (1959a). Consider a planar crack in a body subjected to tensile loading normal to the plane of the crack. Suppose that the material response is linearly elastic, except for a region adjacent to the crack edge where the response departs from linearity. The source of nonlinearity can be plastic deformation, diffuse microcracking, nonlinear interatomic forces, or some other physical mechanism. The crack tip region is said to be *autonomous* at fracture initiation or during crack growth if the following two conditions are met: (i) the extent of the region of nonlinearity from the crack edge is very small compared to all other length dimensions of the body and loading system, and (ii) the mechanical state within this end region at incipient growth or during growth is independent of loading