CHAPTER ONE

Confidence

I. A Decision Problem

You know precious little about the election. It is being held rather far away and it has little national importance. But what you *have* heard – and you haven't heard much – is that the incumbent is ahead of her lone opponent. As a consequence, you find yourself more confident that the incumbent will win. That is, where h is the hypothesis that the incumbent will win, you are more confident that h than you are that $\sim h$.

Not so for g. There is an urn containing exactly 50 black balls and 50 white, all of the same composition and size. The contents of the urn have been thoroughly mixed. One ball has been drawn, but not yet examined. g is the hypothesis that the ball drawn is black. With good reason, you are just as confident that g as you are that $\sim g$.

Suppose I confront you with the following decision problem. Suppose I offer you a choice between

(i) a ticket which entitles you to \$1 if h is true and \$0 if h is false; and
(ii) a coupon which entitles you to \$1 if g is true and \$0 if g is false.

Suppose further that, for the purpose of solving your problem, money is all you intrinsically value and you value every dollar gained or lost equally, no matter what the size of your fortune. You may, if you like, imagine that you have undertaken an obligation to manage the fortune of someone who has these values. In either case, your (her, his) monetary fortune will be to you as pleasure is to a utilitarian. Like pleasure to a utilitarian, it is something measurable, it is something to be maximized, and its quantity in no way affects the value of losing (gaining) one unit of it.

What should you do?

It is not obvious. On the one hand, you are more confident than not that the incumbent will win and you are not more confident than not that the ball drawn is black. This would seem to speak in favor of taking the ticket, of letting the $1 prize ride on *h* rather than on *g*. On the other hand, given how little you know about the election (the evidence you have about its outcome is quite meager) and how much you know about the drawing from the urn (you know enough to know the precise odds that the ball drawn is black), it may seem that, even though you do invest more confidence in *h* than in ~*h*, you have reason to invest more still in *g* and in ~*g*. If so, it would seem that you should take the coupon – you should let the $1 prize ride on *g*.

In fact, you should take the ticket. The nominal purpose of this chapter is to say why, but its more substantial purpose lies elsewhere. As the considerations rehearsed above suggest, what lies at the heart of the decision problem is the question: in which (if either) of *h* and *g* are you warranted in investing more confidence? You will want the $1 to ride on the hypothesis in which you invest more confidence. But, as the considerations rehearsed above also reveal, our epistemic intuitions seem to give us no sure grasp on how that question is to be answered. The purpose of this chapter is to show how, by focusing instead on principles governing rational preference, we can do better. The purpose of this chapter is to provide a first glimpse of how decision theory can constitute a piece of epistemology.

As you can imagine, the reason I mean to offer you for taking the ticket has nothing to do with the physical properties of the ticket or the coupon or the bundles of cash set aside to supply prizes. It has nothing to do with the particular time or location in which the entitlement promised by each option will be conferred, the prize delivered. Rather, it has to do with the state of affairs each option will realize if you choose it. Take the ticket and you will realize the state of affairs in which you increase your fortune by $1 if *h* and by $0 if ~*h* – you will realize ($1 if *h*, $0 if ~*h*). Take the coupon and you will realize the state of affairs in which you increase your fortune by $1 if *g* and by $0 if ~*g* – you will realize ($1 if *g*, $0 if ~*g*). You should take the ticket because you should prefer ($1 if *h*, $0 if ~*h*) to ($1 if *g*, $0 if ~*g*).

I. A DECISION PROBLEM

To convince you of as much, I will devote section II to introducing and defending a set of general principles that purport to describe ways in which the values we are imagining you harbor, together with the demands of reason, would constrain your preferences. In section III, I will show that, on pain of violating these principles, you must prefer (\$1 if *h*, \$0 if ~*h*) to (\$1 if *g*, \$0 if ~*g*). In section IV, I will exhibit how the principles to which I have appealed entail a general (and recognizably Bayesian) theory of rational decision and, with it, an epistemological doctrine I will call "Modest Probabilism." In sections V and VI, I will discuss and evaluate a couple of ways in which the principles in section II might be strengthened, with an eye to their epistemological consequences. Finally, in section VII, I will rehearse, and answer, a number of philosophical objections to Modest Probabilism and to the argument offered on its behalf.

But before I begin, some preliminary remarks are in order.

First, I ask you to imagine throughout that my description of your decision problem is faithful and accurate. That is, I ask you to imagine that there is nothing auspicious about your two options other than what I have explicitly described: the truth-values of the hypotheses involved, *g*, ~*g*, *h* and ~*h*, are not auspicious for you in any way other than the way stated and the truth-values of these hypotheses will not be affected by what option you take or by any of your attitudes. And I ask you to imagine that all my subsequent descriptions of decision problems and states of affairs are faithful and accurate in this way.[2]

Second, I will be assuming, as I talk about decision problems and their elements, that it is tolerably clear what counts as a hypothesis – it is something that is either true or false and not both – and what (by virtue of its vagueness or some other failing) does not count as a hypothesis. People will differ on cases – on what is too vague or unclear to have a truth-value. But I will be happy for my purposes to leave the decision to you. All I will require is that, if *P* and *Q* are members of what, as far as you are concerned, is the set of all hypotheses, then so are all their truth-functional combinations.

Third, I should note at the outset that the particular decision theory I will be placing before you is rather modest in scope. It is

2. Thus, for example, I will be assuming that, if *j* is the hypothesis that you will receive \$5 from your uncle, then (\$100 if *j*, \$0 if ~*j*) includes that \$5 in the \$100 prize you win if *j*.

concerned only with how you should want to constrain your preferences among *well-mannered states of affairs.*

Definition. A is a *well-mannered state of affairs* just in case, for some set of mutually exclusive and jointly exhaustive hypotheses, $\{P_1, \ldots, P_n\}$, and some set of real numbers, $\{a_1, \ldots, a_n\}$, A is identical to ($\$a_1$ if $P_1, \ldots, \$a_n$ if P_n).

[Where all the a_i are equal to the same sum a, we will say that A is *identical to* $\$a$. Thus, for example, we will regard the state of affairs in which you increase your fortune by $1 – a state of affairs we will refer to simply as "$1" – as identical to ($1 if $P_1, \ldots,$ $1 if P_n) where $\{P_1, \ldots, P_n\}$ is any set of mutually exclusive and jointly exhaustive hypotheses.] Moreover, the decision theory is applicable only in contexts in which you have the peculiar values we are assuming you have – i.e., in contexts in which you value only money and every dollar as much as every other no matter what your fortune.

These restrictions (that monetary prizes be in dollars, that the prizes be finite in size, that each state of affairs of interest has but a finite number of possible outcomes, that you care for dollars in the way a utilitarian cares for utility) are designed to make the theory more accessible.[3] Fortunately, as I will explain in section VII, these restrictions will not undermine the generality of the epistemological results whose morals I am concerned to draw. Nor will they in any way hamper my attempt to say how you should solve the decision problem with which we began. We have already supposed that, for the purpose of the problem, you *do* care for dollars the way a utilitarian cares for utility. And, as I have already noted, it is by persuading you that you should prefer the well-mannered state of affairs ($1 if h, $0 if $\sim h$) to the well-mannered state of affairs ($1 if g, $0 if $\sim g$) that I mean to persuade you that you should take the ticket.

II. The Five Principles

Pick any two well-mannered states of affairs, A and B, and the following will be true: either you prefer A to B, you prefer B to A, you are indifferent between A and B or you are undecided between

3. But see section VI for discussion of what happens if we lift the third restriction.

A and *B*. Each of these four states of preference excludes the others. This is particularly important to appreciate in the case of the last two. Both when you are indifferent between *A* and *B* and when you are undecided between *A* and *B* you can be said not to prefer either state of affairs to the other. Nonetheless, indifference and indecision are distinct. When you are indifferent between *A* and *B*, your failure to prefer one to the other is born of a determination that they are equally preferable. When you are undecided, your failure to prefer one to the other is born of no such determination.

Another way to distinguish indifference from indecision is to notice how differently they behave in the context of decision-making. After all, reason subjects your preferences and indifferences to substantive constraint. For example, where *A*, *B* and *C* are any states of affairs, reason holds your preferences and indifferences open to criticism if you are indifferent between *A* and *B* and indifferent between *B* and *C* but you fail to be indifferent between *A* and *C*.[4]

This assumes, of course, that there *is* such a thing as rational indecision. Orthodox Bayesians will demur. They hold that any indecision you may suffer opens your attitudes to criticism – that indecision constitutes a (perhaps excusable, but nonetheless real) failure fully to heed the demands of reason. But, for reasons I will explain in section V, Bayesian orthodoxy is in error. It is an error which condemns orthodox Bayesianism to a false precision that the theory under construction here is designed to avoid.

There is, however, nothing unorthodox in the next three principles. To say that you should want to conform to the first of these principles, is just to spell out in complete generality the moral about rational preference and indifference we have already acknowledged.

1.1 Ordering. Where *A*, *B* and *C* are any well-mannered states of affairs between no pair of which you are undecided,

(i) you do not prefer *A* to *A*; and

4. The demand that your indifferences be transitive is sometimes expressed "If you are indifferent between *A* and *B* and indifferent between *B* and *C*, then you should be indifferent between *A* and *C*." But this is a mistake. Once you harbor the first two indifferences, there are *two* ways to avoid opening your preferences and indifferences to criticism, not one: you can either be indifferent between *A* and *C* or abandon at least one of the first two indifferences. Similar remarks apply to the second clause of Ordering, which I introduce below.

(ii) if you do not prefer A to B and you do not prefer B to C, then you do not prefer A to C.[5]

Now for the second principle. Suppose I had presented you with a somewhat different decision problem than the one which actually confronts you. Suppose I had offered you a choice between \$1 and a stub entitling you to \$2 if *h* is true and \$1.50 if *h* is false. Notice how straightforward the solution would have been. The following argument would have been available to you: "Either *h* is true or *h* is false. If *h* is true and I take the stub I will receive \$2 and will be \$1 better off than I would have been had I taken the \$1. If *h* is false and I take the stub, I will receive \$1.50 and be \$0.50 better off than I would have been had I taken the \$1. So, no matter whether *h* is true or false, my net fortune will be greater if I take the stub. I should prefer taking the stub."

On the other hand, suppose I had offered you a choice between

5. Given that you are undecided between no two of A, B and C, this second clause says that if you either prefer B to A or are indifferent between them and you either prefer C to B or are indifferent between them, then you either prefer C to A or are indifferent between them.

To see how the transitivity of your indifference and preference follows from Ordering, suppose that A, B and C are any well-mannered states of affairs between no pair of which you are undecided. Let "$A \sim B$" signify that you are indifferent between A and B, "$A > B$" that you prefer A to B. Then $A > B$, $B > A$ and $A \sim B$ are mutually exclusive and they exhaust the preferential attitudes you may have toward the pair A and B; in particular, $A \sim B$ iff $A \not> B$ and $B \not> A$.

First, the transitivity of your indifference. Suppose $A \sim B$ and $B \sim C$. Then $A \not> B$ and $B \not> C$ and so, by Ordering, $A \not> C$. Likewise, $C \not> B$ and $B \not> A$ and so, by Ordering, $C \not> A$. But, if $A \not> C$ and $C \not> A$, $A \sim C$. Thus, if $A \sim B$ and $B \sim C$, $A \sim C$.

Now the transitivity of your preference. Suppose $A > B$ and $B > C$. Suppose, for reductio that $A \not> C$. From $A \not> C$ and the supposition that $B > C$ – and hence that $C \not> B$ – it follows by Ordering that $A \not> B$, which contradicts the assumption that $A > B$. So, if $A > B$ and $B > C$, $A > C$.

Some other consequences of Ordering: where A, B, C and D are any well-mannered states of affairs between no pair of which you are undecided, if $A > B$ and $B \sim C$, then $A > C$; if $A \sim B$ and $B > C$, then $A > C$; if $A > B$, $B \sim C$ and $C > D$, then $A > D$.

the stub – it gives you \$2 if *h* is true and \$1.50 if *h* is false – and a coupon which also gives you \$2 if *h* is true and \$1.50 if *h* is false. Again, the solution of your decision problem would have been straightforward. The argument is compelling: "Either *h* is true or *h* is false. If *h* is true, I will be up \$2 no matter whether I take the stub or the coupon. If *h* is false, I will be up \$1.50 no matter whether I take the stub or the coupon. So, no matter whether *h* is true or false, I will do equally well with the stub as I will with the coupon. I should be indifferent between them."

To say that you should want to conform to the next principle is just to generalize these two arguments.

> **1.2 Dominance.** Where A is the well-mannered state of affairs (\$$a_1$ if $P_1, \ldots,$ \$$a_n$ if P_n) and B is the well-mannered state of affairs (\$$b_1$ if $P_1, \ldots,$ \$$b_n$ if P_n),
> (i) if $a_i > b_i$ for every i, then you prefer A to B; and
> (ii) if $a_i = b_i$ for every i, then you are indifferent between A and B.

But now suppose I had begun this chapter with yet a different decision problem. Suppose I had offered you a choice between the ticket and a claim check which entitles you to \$1 if ~*h* and \$0 if *h*. The problem would have been easy to solve. You would have had available to you the following argument. "The choice between the ticket, (\$1 if *h*, \$0 if ~*h*), and the claim check, (\$1 if ~*h*, \$0 if *h*), is just a choice between having \$1 ride on *h* or having \$1 ride on ~*h* – i.e., between identical bets on *h* and on ~*h*. But given that I have good reason to be more confident that *h* is true, I have good reason to be more confident that I will win the bet on *h*. I should take the ticket."

Furthermore, had I placed before you the choice between the ticket and the claim check without giving you any information about the state of your opinion about *h* and ~*h*, and had I promised to offer you a reason why you should take the ticket, you would know that I was promising you a reason why you should be more confident that *h*. For you could reason as follows: "I have a reason to take the ticket only if I have a reason to prefer having \$1 ride on the truth of *h* rather than on its falsehood – i.e., only if I have a reason to find a bet on *h* preferable to an identical bet on ~*h*. And I have a reason to find a bet on *h* preferable to an identical bet on ~*h* only if I have a reason to be more confident that *h* is true."

The moral is that the following relation ought to hold between your states of opinion and your preferences: you find the ticket preferable to the claim check if and only if you are more confident that *h* than you are that ~*h*. This moral can be generalized, by saying that you should want to conform to the following principle.

1.3 Confidence. For any hypotheses *P* and *Q*, you are more confident that *P* than you are that *Q* *if and only if* you prefer ($1 if *P*, $0 if ~*P*) to ($1 if *Q*, $0 if ~*Q*).

But you may worry about this generalization and, in particular, about the necessary condition it would impose on your investing more confidence in *P* than in *Q*. Suppose I have pulled a card from a well-shuffled complete deck of cards and placed it face down on the table. No one has seen what card it is. I then replace the card in the deck and incinerate the deck. Let *p* be 'The card was the ace of spades.' It would seem rational for you to invest more confidence in ~*p* than in *p*. According to Confidence, then, you should also prefer a bet on ~*p* to an identical bet on *p*. But you may wonder how this can be so. There *can be* no bet on either. There is no way to determine whether you win either bet since the evidence relevant to that determination has been destroyed. In general, betting on the truth of hypothesis makes sense only if there is an acknowledged means, agreed upon by the parties to the bet, of determining whether the hypothesis is true or not.

Now this conclusion will not really block my argument that you should take the ticket. After all, the truth-values of *h* and *g* are both determinable. But the conclusion will, if left unchallenged, rob my argument of its generality and philosophical interest. It will place beyond the reach of Confidence a great many of the hypotheses, scientific and otherwise, that we find most interesting. In the case of many hypotheses and theories, there *is* no acknowledged means of determining whether they are true; no one is in a position to make a definitive determination.[6] Fortunately, it is not hard to see why this is a conclusion too hastily drawn.

6. This consideration leads Hilary Putnam (Putnam 1967) to argue that principles linking confidence and betting are incapable of illuminating scientific inquiry.

II. THE FIVE PRINCIPLES

There can be little doubt that, in the case described above, the preconditions for your actually placing a bet on p or on $\sim p$ are simply not met. But then, neither are the preconditions for my having spent the last five minutes on Pinney's Beach in Nevis. I have, in fact, spent the last five minutes working in Milwaukee and there is nothing anyone can do to change that. Yet it still makes sense for me to prefer having spent the last five minutes on that idyllic Caribbean beach to having spent the time at work in Milwaukee. But if it ever makes sense to prefer that things were otherwise than they in fact were, we must reject the view that the preconditions for the realization of a state of affairs must be met for there to be sense in speaking of your preferring that state of affairs to some other. Once we reject that view, it is hard to see anything at all problematic in supposing – even as we concede that the preconditions for betting on p and for betting on $\sim p$ are not met – that you prefer having \$1 ride on $\sim p$ to having \$1 ride on p.

Obviously, Dominance and Confidence depend for their propriety on the assumption that you value only money. The next principle exploits, in addition, the assumption that you value every dollar as much as every other no matter what your fortune.

As should be clear by now, the well-mannered states of affairs over which the foregoing principles constrain your preferences are *types* of states of affairs of which their realizations are *tokens*.[7] As a type, the state of affairs in which you increase your fortune by \$1 if h and by \$0 if $\sim h$, is unique. Yet it admits of many tokens, each of which is a realization of that state of affairs at a particular time. (Your taking the ticket would constitute one such realization.) Now let us say that

> **Definition.** A sequence of well-mannered states of affairs is *realized* just when each state of affairs that occurs in the sequence is realized as many times as it occurs in the sequence.

Thus, to realize the sequence

(1) (\$1 if h, \$0 if $\sim h$), (\$1 if h, \$0 if $\sim h$), (\$1 if g, \$0 $\sim g$),

7. For example, Dominance says, "Where A is *the* well-mannered state of affairs, (\$$a_1$ if P_1,..., \$$a_n$ if P_n)..." not "Where A is *a* well-mannered state of affairs, (\$$a_1$ if P_1,..., \$$a_n$ if P_n)...."

is to realize (\$1 if h, \$0 if $\sim h$) twice and (\$1 if g, \$0 $\sim g$) once. Next, let us say that

> **Definition.** A sequence of well-mannered states of affairs ϕ is a *decomposition of* a well-mannered state of affairs A (*A is composable from ϕ*) just in case ϕ is finite and it is logically impossible that the realization of ϕ will effect (*qua* realization)[8] a different net change in your fortune than the realization of A will.

Thus the sequence (1) constitutes a decomposition of

(2) (\$3 if h & g, \$2 if h & $\sim g$, \$1 if $\sim h$ & g, \$0 if $\sim h$ & $\sim g$).

Finally, let us say that

> **Definition.** You *place a monetary value of \$a on A* just if you are indifferent between \$$a$ and A.

The fourth principle says the following:

> **1.4 Decomposition.** *If*
> (i) A is a well-mannered state of affairs;
> (ii) ϕ is a decomposition of A; and
> (iii) you place a monetary value on A and on each of the terms of ϕ;
>
> *then* the value you place on A is equal to the sum of the values you place on the terms of ϕ.

Thus Decomposition requires that the sum of the monetary values you place on the terms of (1) equal the monetary value you place on (2).

But why should you want to submit to this requirement? Suppose you were bound by a budgetary constraint (or an aversion to gambling) that forbids you to gamble more than \$1. Why could you not then rationally place a monetary value of \$0.60 on each of the first two terms of (1) and a value of \$0.50 on the last yet be unwilling to place a monetary value equal to their sum, \$1.70, on (2)?

It is because of the way we are supposing you value money. We are supposing that you value only money, and every dollar as much as every other no matter what your fortune. It is incompatible with

8. That is, apart from what other consequences the realization of the sequence might have.