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*Orthonormal Systems and
Banach Space Geometry*

ALBRECHT PIETSCH & JÖRG WENZEL
University of Jena



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Preface

This book is based on the pioneering work of (in chronological order)
R. C. James, S. Kwapien, B. Maurey, G. Pisier, D. L. Burkholder and
J. Bourgain.

We have done our best to unify and simplify the material. All participants of the Jenaer Seminar '*Operatorenideale*' contributed their ideas and their patience. Above all, we are indebted to A. Hinrichs who made several significant improvements. From S. Geiss we learnt many results and techniques related to the theory of martingales. Particular gratitude goes to H. Jarchow (Zürich) for various helpful remarks.

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