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978-0-521-62349-0 - Geometry of Sporadic Groups II: Representations and Amalgams

A. A. Ivanov and S. V. Shpectorov

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