

*Introduction to High Energy Physics*  
*4th Edition*

Donald H. Perkins  
*University of Oxford*



**CAMBRIDGE**  
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, UK  
40 West 20th Street, New York, NY 10011-4211, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
Ruiz de Alarcón 13, 28014, Madrid, Spain  
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

Fourth edition © Donald H. Perkins 2000

This book is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without  
the written permission of Cambridge University Press.

First published by Addison-Wesley Publishing Company Inc. 1972  
Fourth edition first published by Cambridge University Press 2000  
Reprinted 2001, 2003

Printed in the United Kingdom at the University Press, Cambridge

*Typeface* Times 11/14pt. *System* L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub> [DBD]

*A catalogue record of this book is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Perkins, Donald H.  
Introduction to high energy physics / Donald H. Perkins. – 4th ed.  
p. cm. ISBN 0 521 62196 8 (hc.)  
1. Particles (Nuclear physics) I. Title.  
QC93.2.P47 1999  
529.7'2–dc21 98-51723 CIP

ISBN 0 521 62196 8 hardback

# Contents

<i>Preface</i>	<i>page xi</i>
<b>1 Quarks and leptons</b>	<b>1</b>
1.1 Preamble	1
1.2 The Standard Model of particle physics	7
1.3 Particle classification: fermions and bosons	12
1.4 Particles and antiparticles	13
1.5 Free particle wave equations	16
1.6 Helicity states: helicity conservation	19
1.7 Lepton flavours	20
1.8 Quark flavours	22
1.9 The cosmic connection	26
Problems	33
<b>2 Interactions and fields</b>	<b>35</b>
2.1 Classical and quantum pictures of interactions	35
2.2 The Yukawa theory of quantum exchange	36
2.3 The boson propagator	37
2.4 Feynman diagrams	38
2.5 Electromagnetic interactions	40
2.6 Renormalisation and gauge invariance	42
2.7 Strong interactions	43
2.8 Weak and electroweak interactions	46
2.9 Gravitational interactions	51
2.10 The interaction cross-section	51
2.11 Decays and resonances	55
Problems	61

<b>3</b>	<b>Invariance principles and conservation laws</b>	<b>63</b>
3.1	Translation and rotation operators	63
3.2	The parity operation	65
3.3	Pion spin and parity	66
3.4	Parity of particles and antiparticles	69
3.5	Tests of parity conservation	72
3.6	Charge conjugation invariance	73
3.7	Charge conservation and gauge invariance	75
3.8	Baryon and lepton conservation	79
3.9	<i>CPT</i> invariance	81
3.10	<i>CP</i> violation and <i>T</i> violation	81
3.11	Neutron electric dipole moment	83
3.12	Isospin symmetry	87
3.13	Isospin in the two-nucleon and pion–nucleon systems	88
3.14	Isospin, strangeness and hypercharge	91
	Problems	93
<b>4</b>	<b>Quarks in hadrons</b>	<b>95</b>
4.1	Charm and beauty; the heavy quarkonium states	95
4.2	Comparison of quarkonium and positronium levels	102
4.3	The baryon decuplet	109
4.4	Quark spin and colour	114
4.5	The baryon octet	115
4.6	Quark–antiquark combinations: the light pseudoscalar mesons	118
4.7	The light vector mesons	121
4.8	Other tests of the quark model	123
4.9	Mass relations and hyperfine interactions	126
4.10	Electromagnetic mass differences and isospin symmetry	129
4.11	Magnetic moments of baryons	130
4.12	Mesons built of light and heavy quarks	132
4.13	The top quark	134
	Problems	139
<b>5</b>	<b>Lepton and quark scattering</b>	<b>140</b>
5.1	The process $e^+e^- \rightarrow \mu^+\mu^-$	140
5.2	$e^+e^-$ annihilation to hadrons	144
5.3	Electron–muon scattering, $e^-\mu^+ \rightarrow e^-\mu^+$	147
5.4	Neutrino–electron scattering, $\nu_e e \rightarrow \nu_e e$	150
5.5	Elastic lepton–nucleon scattering	154
5.6	Deep inelastic scattering and partons	155
5.7	Deep inelastic scattering and quarks	159

5.8	Experimental results on quark distributions in the nucleon	162
5.9	Sum rules	166
5.10	Summary	168
	Problems	168
<b>6</b>	<b>Quark interactions and QCD</b>	<b>171</b>
6.1	The colour quantum number	171
6.2	The QCD potential at short distances	172
6.3	The QCD potential at large distances: the string model	178
6.4	Gluon jets in $e^+e^-$ annihilation	180
6.5	Running couplings in QED and QCD	181
6.6	Evolution of structure functions in deep inelastic scattering	186
6.7	Gluonium and the quark–gluon plasma	190
	Problems	192
<b>7</b>	<b>Weak interactions</b>	<b>194</b>
7.1	Classification	194
7.2	Lepton universality	195
7.3	Nuclear $\beta$ -decay: Fermi theory	197
7.4	Inverse $\beta$ -decay: neutrino interactions	201
7.5	Parity nonconservation in $\beta$ -decay	202
7.6	Helicity of the neutrino	205
7.7	The $V - A$ interaction	206
7.8	Conservation of weak currents	209
7.9	The weak boson and Fermi couplings	210
7.10	Pion and muon decay	210
7.11	Neutral weak currents	213
7.12	Observation of $W^\pm$ and $Z^0$ bosons in $p\bar{p}$ collisions	215
7.13	$Z^0$ production at $e^+e^-$ colliders	220
7.14	Weak decays of quarks. The GIM model and the CKM matrix	221
7.15	Neutral $K$ mesons	226
7.16	$CP$ violation in the neutral kaon system	232
7.17	Cosmological $CP$ violation	237
7.18	$D^0-\bar{D}^0$ and $B^0-\bar{B}^0$ mixing	238
	Problems	239
<b>8</b>	<b>Electroweak interactions and the Standard Model</b>	<b>242</b>
8.1	Introduction	242
8.2	Divergences in the weak interactions	243
8.3	Introduction of neutral currents	245
8.4	The Weinberg–Salam model	246

8.5	Intermediate boson masses	248
8.6	Electroweak couplings of leptons and quarks	249
8.7	Neutrino scattering via $Z$ exchange	250
8.8	Asymmetries in the scattering of polarised electrons by deuterons	253
8.9	Observations on the $Z$ resonance	255
8.10	Fits to the Standard Model and radiative corrections	260
8.11	$W$ pair production	262
8.12	Spontaneous symmetry breaking and the Higgs mechanism	263
8.13	Higgs production and detection	271
	Problems	274
<b>9</b>	<b>Physics beyond the Standard Model</b>	<b>276</b>
9.1	Supersymmetry	277
9.2	Grand unified theories: the $SU(5)$ GUT	278
9.3	Unification energy and weak mixing angle	280
9.4	Supersymmetric $SU(5)$	282
9.5	Proton decay	282
9.6	Neutrino mass: Dirac and Majorana neutrinos	284
9.7	Neutrino oscillations	287
9.8	Magnetic monopoles	299
9.9	Superstrings	300
	Problems	301
<b>10</b>	<b>Particle physics and cosmology</b>	<b>303</b>
10.1	Hubble's law and the expanding universe	303
10.2	Friedmann equation	304
10.3	Cosmic microwave radiation: the hot Big Bang	307
10.4	Radiation and matter eras	311
10.5	Nucleosynthesis in the Big Bang	313
10.6	Baryon–antibaryon asymmetry	317
10.7	Dark matter	319
10.8	Inflation	326
10.9	Neutrino astronomy: SN 1987A	330
	Problems	336
<b>11</b>	<b>Experimental methods</b>	<b>338</b>
11.1	Accelerators	338
11.2	Colliding-beam accelerators	343
11.3	Accelerator complexes	346
11.4	Secondary particle separators	346
11.5	Interaction of charged particles and radiation with matter	349

11.6	Detectors of single charged particles	355
11.7	Shower detectors and calorimeters	368
	Problems	375
<i>Appendix A</i>	<i>Table of elementary particles</i>	377
<i>Appendix B</i>	<i>Milestones in particle physics</i>	379
<i>Appendix C</i>	<i>Clebsch–Gordan coefficients and d-functions</i>	386
<i>Appendix D</i>	<i>Spherical harmonics, d-functions and Clebsch–Gordan coefficients</i>	393
<i>Appendix E</i>	<i>Relativistic normalisation of cross-sections and decay rates</i>	396
	<i>Glossary</i>	398
	<i>Answers to problems</i>	408
	<i>Bibliography</i>	412
	<i>References</i>	418
	<i>Index</i>	421

# 1

## Quarks and leptons

### 1.1 Preamble

The subject of elementary particle physics may be said to have begun with the discovery of the electron 100 years ago. In the following 50 years, one new particle after another was discovered, mostly as a result of experiments with cosmic rays, the only source of very high energy particles then available. However, the subject really blossomed after 1950, following the discovery of new elementary particles in cosmic rays; this stimulated the development of high energy accelerators, providing intense and controlled beams of known energy that were finally to reveal the quark substructure of matter and put the subject on a sound quantitative basis.

#### 1.1.1 Why high energies?

Particle physics deals with the study of the elementary constituents of matter. The word ‘elementary’ is used in the sense that such particles have no known structure, i.e. they are pointlike. How pointlike is pointlike? This depends on the spatial resolution of the ‘probe’ used to investigate possible structure. The resolution is  $\Delta r$  if two points in an object can just be resolved as separate when they are a distance  $\Delta r$  apart. Assuming the probing beam itself consists of pointlike particles, the resolution is limited by the de Broglie wavelength of these particles, which is  $\lambda = h/p$  where  $p$  is the beam momentum and  $h$  is Planck’s constant. Thus beams of high momentum have short wavelengths and can have high resolution. In an optical microscope, the resolution is given by

$$\Delta r \simeq \lambda / \sin \theta$$

where  $\theta$  is the angular aperture of the light beam used to view the structure of an object. The object scatters light into the eyepiece, and the larger the angle of scatter  $\theta$  and the smaller the wavelength  $\lambda$  of the incident beam the better is the resolution. For example an ultraviolet microscope has better resolution and reveals



more detail than one using visible light. Substituting the de Broglie relation, the resolution becomes

$$\Delta r \simeq \frac{\lambda}{\sin \theta} = \frac{h}{p \sin \theta} \simeq \frac{h}{q}$$

so that  $\Delta r$  is inversely proportional to the momentum  $q$  transferred to the photons, or other particles in an incident beam, when these are scattered by the target.<sup>†</sup> Thus a value of momentum transfer such that  $qc = 10 \text{ GeV} = 10^{10} \text{ eV}$  – easily attainable with present accelerator beams – gives a spatial resolution  $hc/(qc) \sim 10^{-16} \text{ m}$ , about 10 times smaller than the known radius of the charge and mass distribution of a proton (see Table 1.1 for the values of the units employed).

In the early decades of the twentieth century, particle-beam energies from accelerators reached only a few MeV ( $10^6 \text{ eV}$ ), and their resolution was so poor that protons and neutrons could themselves be regarded as elementary and pointlike. At the present day, with a resolution thousands of times better, the fundamental pointlike constituents of matter appear to be quarks and leptons, which are the main subject of this text. Of course, it is possible that they in turn may have an inner structure, but there is no present evidence for this, and whether they do will be for future experiments to decide.

The second reason for high energies in experimental particle physics is simply that many of the elementary particles are extremely massive and the energy  $mc^2$  required to create them is correspondingly large. The heaviest elementary particle detected so far, the ‘top’ quark (which has to be created as a pair with its antiparticle) has  $mc^2 \simeq 175 \text{ GeV}$ , nearly 200 times the mass–energy of a proton.

At this point it should be mentioned that the total energy in accelerator beams required to create such massive particles in sufficient intensities is quite substantial. For example, an energy per particle of 1 TeV ( $10^{12} \text{ eV}$ ) in beams consisting of bunches of  $10^{13}$  accelerated particles every second will correspond to a total kinetic energy in each bunch of 1.6 megajoules, equal to the energy of 30 000 light bulbs, or of a 15 tonne truck travelling at 30 mph.

### 1.1.2 Units in high energy physics

The basic units in physics are length, mass and time and the SI system expresses these in metres, kilograms and seconds. Such units are not very appropriate in high energy physics, where typical lengths are  $10^{-15} \text{ m}$  and typical masses are  $10^{-27} \text{ kg}$ .

Table 1.1 summarises the units commonly used in high energy physics. The unit of length is the *femtometre* or *fermi*, where  $1 \text{ fm} = 10^{-15} \text{ m}$ ; for example, the root mean square radius of the charge distribution of a proton is 0.8 fm. The

<sup>†</sup> To be exact, in an elastic collision with a massive target, the momentum transfer will be  $q = 2p \sin(\theta/2)$ , if  $\theta$  is the angle of deflection.

Table 1.1. *Units in high energy physics*

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	$10^{-15}$ m
energy	1 GeV = $10^9$ eV	$1.602 \times 10^{-10}$ J
mass, $E/c^2$	1 GeV/ $c^2$	$1.78 \times 10^{-27}$ kg
$\hbar = h/(2\pi)$	$6.588 \times 10^{-25}$ GeV s	$1.055 \times 10^{-34}$ J s
$c$	$2.998 \times 10^{23}$ fm s $^{-1}$	$2.998 \times 10^8$ m s $^{-1}$
$\hbar c$	0.1975 GeV fm	$3.162 \times 10^{-26}$ J m

(b)

natural units, $\hbar = c = 1$		
mass, $Mc^2/c^2$	1 GeV	
length, $\hbar c/(Mc^2)$	1 GeV $^{-1}$ = 0.1975 fm	
time, $\hbar c/(Mc^3)$	1 GeV $^{-1}$ = $6.59 \times 10^{-25}$ s	
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$		
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$	
Relations between energy units		
1 MeV = $10^6$ eV	1 GeV = $10^3$ MeV	1 TeV = $10^3$ GeV

commonly used unit of energy is the GeV, convenient because it is typical of the mass–energy  $mc^2$  of strongly interacting particles. For example, a proton has  $M_p c^2 = 0.938$  GeV.

In calculations, the quantities  $\hbar = h/(2\pi)$  and  $c$  occur frequently, sometimes to high powers, and it is advantageous to use units in which we set  $\hbar = c = 1$ . Having chosen these two units, we are still at liberty to specify one more unit, e.g. the unit of energy, and the common choice, as indicated above, is the GeV. With  $c = 1$  this is also the mass unit. As shown in the table, the unit of length will then be 1 GeV $^{-1}$  = 0.197 fermi, while the corresponding unit of time is 1 GeV $^{-1}$  =  $6.59 \times 10^{-25}$  s.

Throughout this text we shall be dealing with interactions between charges – which can be the familiar electric charge of electromagnetic interactions, the strong charge of the strong interaction or the weak charge of the weak interaction. In the SI system the unit electric charge,  $e$ , is measured in coulombs and the fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

Here  $\epsilon_0$  is the permittivity of free space, while its permeability is defined as  $\mu_0$ ,

such that  $\epsilon_0\mu_0 = 1/c^2$ . For interactions in general, such units are not useful and we can define  $e$  in Heaviside–Lorentz units, which require  $\epsilon_0 = \mu_0 = \hbar = c = 1$ , so that

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

with similar definitions that relate charges and coupling constants analogous to  $\alpha$  in the other interactions.

### 1.1.3 Relativistic transformations

In most of the processes to be considered in high energy physics, the individual particles have relativistic or near relativistic velocities,  $v \sim c$ . This means that the result of a measurement, e.g. the lifetime of an unstable particle, will depend on the reference frame in which it is made. It follows that one requirement of any theory of elementary particles is that it should obey a fundamental symmetry, namely invariance under a relativistic transformation, so that the equations will have the same form in all reference frames. This can be achieved by formulating the equations in terms of 4-vectors, which we now discuss briefly, together with the notation employed in this text.

The relativistic relation between total energy  $E$ , the vector 3-momentum  $\mathbf{p}$  (with Cartesian components  $p_x, p_y, p_z$ ) and the rest mass  $m$  for a free particle is

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4$$

or, in units with  $c = 1$

$$E^2 = \mathbf{p}^2 + m^2$$

The components  $p_x, p_y, p_z, E$  can be written as components of an energy–momentum 4-vector  $p_\mu$ , where  $\mu = 1, 2, 3, 4$ . In the Minkowski convention used in this text, the three momentum (or space) components are taken to be real and the energy (or time) component to be imaginary, as follows:

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$$

so that

$$p^2 = \sum_{\mu} p_{\mu}^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = \mathbf{p}^2 - E^2 = -m^2 \quad (1.1)$$

Thus  $p^2$  is a relativistic invariant. Its value is  $-m^2$ , where  $m$  is the rest mass, and clearly has the same value in all reference frames. If  $E, \mathbf{p}$  refer to the values measured in the lab frame  $\Sigma$  then those in another frame, say  $\Sigma'$ , moving along

the  $x$ -axis with velocity  $\beta c$  are found from the Lorentz transformation, given in matrix form by

$$p'_\mu = \sum_{\nu=1}^4 \alpha_{\mu\nu} p_\nu$$

where

$$\alpha_{\mu\nu} = \begin{vmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{vmatrix}$$

and  $\gamma = 1/\sqrt{1 - \beta^2}$ . Thus

$$\begin{aligned} p'_1 &= \gamma p_1 + i\beta\gamma p_4 \\ p'_2 &= p_2 \\ p'_3 &= p_3 \\ p'_4 &= -i\beta\gamma p_1 + \gamma p_4 \end{aligned}$$

In terms of energy and momentum

$$\begin{aligned} p'_x &= \gamma(p_x - \beta E) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - \beta p_x) \end{aligned}$$

with, of course,  $\mathbf{p}'^2 - E'^2 = -m^2$ . The above transformations apply equally to the space-time coordinates, making the replacements  $p_1 \rightarrow x_1 (= x)$ ,  $p_2 \rightarrow x_2 (= y)$ ,  $p_3 \rightarrow x_3 (= z)$  and  $p_4 \rightarrow x_4 (= it)$ .

The 4-momentum squared in (1.1) is an example of a Lorentz scalar, i.e. the invariant scalar product of two 4-vectors,  $\sum p_\mu p_\mu$ . Another example is the phase of a plane wave, which determines whether it is at a crest or a trough and which must be the same for all observers. With  $\mathbf{k}$  and  $\omega$  as the propagation vector and the angular frequency, and in units  $\hbar = c = 1$ ,

$$\text{phase} = \mathbf{k} \cdot \mathbf{x} - \omega t = \mathbf{p} \cdot \mathbf{x} - Et = \sum p_\mu x_\mu$$

The Minkowski notation used here for 4-vectors defines the *metric*, namely the square of the 4-vector momentum  $p = (\mathbf{p}, iE)$  so that

$$\text{metric} = (4\text{-momentum})^2 = (3\text{-momentum})^2 - (\text{energy})^2$$

In analogy with the space-time components, the components  $p_{x,y,z}$  of 3-momentum are said to be *spacelike* and the energy component  $E$ , *timelike*. Thus,

if  $q$  denotes the 4-momentum transfer in a reaction, i.e. is  $q = p - p'$  where  $p, p'$  are the initial and final 4-momenta, then

$$\begin{aligned} q^2 > 0 & \text{ is spacelike, e.g. in a scattering process} \\ q^2 < 0 & \text{ is timelike, e.g. the squared mass of a free particle} \end{aligned} \quad (1.2)$$

A different notation is used in texts on field theory. These avoid the use of the imaginary fourth component ( $p_4 = iE$ ) and introduce the negative sign via the metric tensor  $g_{\mu\nu}$ . The scalar product of 4-vectors  $A$  and  $B$  is then defined as

$$AB = g_{\mu\nu} A_\mu B_\nu = A_0 B_0 - \mathbf{A} \cdot \mathbf{B} \quad (1.3)$$

where all the components are real. Here  $\mu, \nu = 0$  stand for the energy (or time) component and  $\mu, \nu = 1, 2, 3$  for the momentum (or space) components, and

$$g_{00} = +1, \quad g_{11} = g_{22} = g_{33} = -1, \quad g_{\mu\nu} = 0 \quad \text{for } \mu \neq \nu \quad (1.4)$$

This metric results in Lorentz scalars with sign opposite to those using the Minkowski convention in (1.2), so that a spacelike (or timelike) 4-momentum has  $q^2 < 0$  (or  $q^2 > 0$ ) respectively. Sometimes, to avoid writing negative quantities, re-definitions have to be made. In deep inelastic electron scattering,  $q^2$  is spacelike and negative, as defined in (1.3), and in discussing such processes it has become common to define the positive quantity  $Q^2 = -q^2$ . This simply illustrates the fact that the sign of the metric is just a matter of convention and does not in any way affect the physical results.

#### 1.1.4 Fixed-target and colliding beam accelerators

As an example of the application of 4-vector notation, we consider the energy available for particle creation in fixed-target and in colliding-beam accelerators (see also Chapter 11).

Suppose an incident particle of mass  $m_A$ , total energy  $E_A$  and momentum  $\mathbf{p}_A$  hits a target particle of mass  $m_B$ , energy  $E_B$ , momentum  $\mathbf{p}_B$ . The total 4-momentum, squared, of the system is

$$p^2 = (\mathbf{p}_A + \mathbf{p}_B)^2 - (E_A + E_B)^2 = -m_A^2 - m_B^2 + 2\mathbf{p}_A \cdot \mathbf{p}_B - 2E_A E_B \quad (1.5)$$

The centre-of-momentum system (cms) is defined as the reference frame in which the total 3-momentum is zero. If the total energy in the cms is denoted  $E^*$ , then we also have  $p^2 = -E^{*2}$ .

Suppose first of all that the target particle ( $m_B$ ) is at rest in the laboratory (lab) system, so that  $\mathbf{p}_B = 0$  and  $E_B = m_B$ , while  $E_A$  is the energy of the incident particle in the lab system. Then

$$E^{*2} = -p^2 = m_A^2 + m_B^2 + 2m_B E_A \quad (1.6)$$

Secondly, suppose that the incident and target particles travel in opposite directions, as would be the case in an  $e^+e^-$  or a  $p\bar{p}$  collider. Then, with  $p_A$  and  $p_B$  denoting the absolute values of the 3-momenta, the above equation gives

$$\begin{aligned} E^{*2} &= -p^2 = 2(E_A E_B + p_A p_B) + (m_A^2 + m_B^2) \\ &\simeq 4E_A E_B \end{aligned} \quad (1.7)$$

if  $m_A, m_B \ll E_A, E_B$ . This result is for a head-on collision. For two beams crossing at an angle  $\theta$ , the result would be  $E^{*2} = 2E_A E_B(1 + \cos\theta)$ . We note that the cms energy available for new particle creation in a collider with equal energies  $E$  in the two beams rises linearly with  $E$ , i.e.  $E^* \simeq 2E$ , while for a fixed-target machine the cms energy rises as the square root of the incident energy,  $E^* \simeq \sqrt{2m_B E_A}$ . Obviously, therefore, the highest possible energies for creating new particles are to be found at colliding-beam accelerators. As an example, the cms energy of the Tevatron  $p\bar{p}$  collider at Fermilab is  $E^* = 2 \text{ TeV} = 2000 \text{ GeV}$ . To obtain the same cms energy with a fixed-target accelerator, the energy of the proton beam, in collision with a target nucleon, would have to be  $E_A = E^{*2}/(2m_B) \simeq 2 \times 10^6 \text{ GeV} = 2000 \text{ TeV}$ .

## 1.2 The Standard Model of particle physics

### 1.2.1 The fundamental fermions

Practically all experimental data from high energy experiments can be accounted for by the so-called *Standard Model* of particles and their interactions, formulated in the 1970s. According to this model, all matter is built from a small number of fundamental spin  $\frac{1}{2}$  particles, or *fermions*: six *quarks* and six *leptons*. For each of the various fundamental constituents, its symbol and the ratio of its electric charge  $Q$  to the elementary charge  $e$  of the electron are given in Table 1.2.

The *leptons* carry integral electric charge. The electron  $e$  with unit negative charge is familiar to everyone, and the other charged leptons are the muon  $\mu$  and the tauon  $\tau$ . These are heavy versions of the electron. The neutral leptons are called *neutrinos*, denoted by the generic symbol  $\nu$ . A different ‘flavour’ of neutrino is paired with each ‘flavour’ of charged lepton, as indicated by the subscript. For example, in nuclear  $\beta$ -decay, an electron  $e$  is emitted together with an electron-type neutrino,  $\nu_e$ . The charged muon and tauon are both unstable, and decay spontaneously to electrons, neutrinos and other particles. The mean lifetime of the muon is  $2.2 \times 10^{-6} \text{ s}$ , that of the tauon only  $2.9 \times 10^{-13} \text{ s}$ .

Neutrinos were postulated by Pauli in 1930 in order to account for the energy and momentum missing in the process of nuclear  $\beta$ -decay (see Figure 1.1). The actual existence of neutrinos as independent particles, detected by their interactions, was

Table 1.2. *The fundamental fermions*

Particle	Flavour			$Q/ e $
leptons	$e$	$\mu$	$\tau$	$-1$
	$\nu_e$	$\nu_\mu$	$(\nu_\tau)$	$0$
quarks	$u$	$c$	$t$	$+\frac{2}{3}$
	$d$	$s$	$b$	$-\frac{1}{3}$

first demonstrated in 1956. The tau neutrino is shown in parentheses because its interactions have not so far (1999) been observed.

The *quarks* carry fractional charges, of  $+\frac{2}{3}|e|$  or  $-\frac{1}{3}|e|$ . In the table, the quark masses increase from left to right, just as they do for the leptons (see Tables 1.4 and 1.5). And, just as for the leptons, the quarks are grouped into pairs differing by one unit of electric charge. The quark type or ‘flavour’ is denoted by a symbol:  $u$  for ‘up’,  $d$  for ‘down’,  $s$  for ‘strange’,  $c$  for ‘charmed’,  $b$  for ‘bottom’ and  $t$  for ‘top’. How did such odd names get chosen? The ‘ $s$  for strange’ quark terminology came about because these quarks turned out to be constituents of the so-called ‘strange particles’ discovered in cosmic rays (long before quarks were postulated). Their behaviour was strange in the sense that they were produced prolifically in strong interactions, and therefore would be expected to decay on a strong interaction timescale ( $10^{-23}$  s); instead they decayed extremely slowly, by weak interactions. The solution to this puzzle was that these particles carried a new quantum number,  $S$  for strangeness, conserved in strong interactions – so that they were always produced in pairs with  $S = +1$  and  $S = -1$  but they decayed singly and weakly, with a change in strangeness,  $\Delta S = \pm 1$ , into non-strange particles (see Figure 1.10). The choice of the name ‘ $c$  for charm’ was perhaps a reaction to strangeness, while ‘top’ and ‘bottom’ are logical names for the partners of up and down quarks. In turn, the up and down quarks were so named because of isospin symmetry (see Section 3.12), according to which each possesses one of the two components  $\pm\frac{1}{2}$  of an isospin vector of value  $I = \frac{1}{2}$ , which, like a spin vector, can point ‘up’ or ‘down’.

While leptons exist as free particles, quarks seem not to do so. It is a peculiarity of the strong forces between the quarks that they can be found only in combinations such as  $uud$ , not singly. This phenomenon of quark confinement is, even today, not properly understood.

Protons and neutrons consist of the lightest  $u$  and  $d$  quarks, three at a time: a proton consists of  $uud$ , a neutron consists of  $ddu$ . The common material of the present universe is the stable particles, i.e. the electrons  $e$  and the  $u$  and

$d$  quarks. The heavier quarks  $s, c, b, t$  also combine to form particles akin to, but much heavier than, the proton and neutron, but these are unstable and decay rapidly (in typically  $10^{-13}$  s) to  $u, d$  combinations, just as the heavy leptons decay to electrons. Only in very high energy collisions at man-made accelerators, or naturally in cosmic rays, are the heavy, unstable varieties observed.

Table 1.2 shows that the three lepton pairs are exactly matched by the three quark pairs. As we shall see later, it is necessary to introduce a further degree of freedom for the quarks: each flavour of quark comes in three different *colours* (the word ‘colour’ is simply a name to distinguish the three types). If we allow for three colours, the total charge of the  $u, c, t$  quarks is  $3 \times 3 \times \frac{2}{3} = 6|e|$ , that of the  $d, s, b$  quarks is  $-3 \times 3 \times \frac{1}{3} = -3|e|$  and that of the leptons is  $-3 \times 1|e| = -3|e|$ . The total charge of all the fermions is then zero. This is the actual condition that the Standard Model should be free of so-called ‘anomalies’ and is a renormalisable field theory. It is also, it turns out, a property of the grand unified theories that unify the strong, electromagnetic and weak interactions at very high energies, as described in Chapter 9.

### 1.2.2 The interactions

We have looked at the particles; the Standard Model also comprises their interactions. As we discuss in the next chapter, the different interactions are described in quantum language in terms of the exchange of characteristic *bosons* (particles of integral spin) between the fermion constituents. These boson mediators are listed in Table 1.3.

There are four types of fundamental interaction or field, as follows.

*Strong* interactions are responsible for binding the quarks in the neutron and proton, and the neutrons and protons within nuclei. The interquark force is mediated by a massless particle, the *gluon*.

*Electromagnetic* interactions are responsible for virtually all the phenomena in extra-nuclear physics, in particular for the bound states of electrons with nuclei, i.e. atoms and molecules, and for the intermolecular forces in liquids and solids. These interactions are mediated by *photon* exchange.

*Weak* interactions are typified by the slow process of nuclear  $\beta$ -decay, involving the emission by a radioactive nucleus of an electron and neutrino. The mediators of the weak interactions are the  $W^\pm$  and  $Z^0$  bosons, with masses of order 100 times the proton mass.

*Gravitational* interactions act between all types of particle. On the scale of experiments in particle physics, gravity is by far the weakest of all the fundamental interactions, although of course it is dominant on the scale of the universe. It is supposedly mediated by exchange of a spin 2 boson, the *graviton*. Very refined



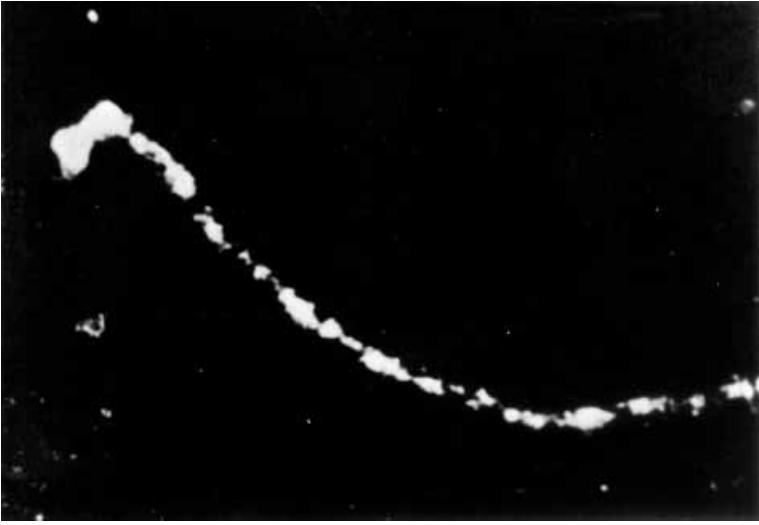


Fig. 1.1. Cloud chamber photograph of the birth of an antineutrino. It depicts the  $\beta$ -decay of the radioactive nucleus  ${}^6\text{He} \rightarrow {}^6\text{Li} + e^- + \bar{\nu}_e + 3.5 \text{ MeV}$ . The long track is that of the electron, the short thick track that of the recoiling  ${}^6\text{Li}$  nucleus. Some momentum is missing, and has to be ascribed to an uncharged particle (an antineutrino) travelling upwards in the picture (after Csikay and Szalay 1957). The cloud chamber consists essentially of a glass-fronted cylindrical tank of gas saturated with water vapour. Upon applying a sudden expansion by means of a piston at the rear of the chamber, the gas cools adiabatically and becomes supersaturated. Water vapour therefore condenses as droplets, preferentially upon charged ions created, for example, by the passage of a charged particle through the gas. The cloud chamber was invented by C.T.R. Wilson for a quite different purpose: to try to reproduce, in the laboratory, the ‘glory’ phenomenon he had observed on a Scottish mountain top. Wilson failed in this endeavour but by 1912 had given the world a valuable new technique for nuclear research.

Table 1.3. *The boson mediators*

Interaction	Mediator	Spin/parity
strong	gluon, $G$	$1^-$
electromagnetic	photon, $\gamma$	$1^-$
weak	$W^\pm, Z^0$	$1^-, 1^+$
gravity	graviton, $g$	$2^+$

experiments to detect gravitons (en masse, as gravitational waves) are currently under way.

To have four independent and apparently unrelated interaction fields is rather unsatisfactory, and physicists from Einstein onwards have speculated that the

different interactions are different aspects of a single, unified field. In the 1970s, experiments showed that the weak and electromagnetic interactions can indeed be unified, and would have the same strength at very high energies; only at lower energies is the symmetry broken so that their apparent strengths are very different. Thus some progress has been made and more is expected.

We shall be discussing these interactions in more detail in Chapter 2, but simply remark here that, in the everyday world, all four are of importance. The physical processes in the Sun – our chief source of energy on Earth – provide a good illustration. The Sun originally condensed under gravity from a cloud of hydrogen, until the core reached by compression a high enough temperature ( $10^7$  degrees) for thermonuclear fusion reactions to begin. In these reactions hydrogen is converted to helium. The first-stage reaction is actually a weak interaction,  $p + p \rightarrow d + e^+ + \nu_e$ : two protons fuse to form a deuteron  $d$ , a positron  $e^+$  and a neutrino  $\nu_e$ . As described in subsection 9.7.1, these neutrinos have been detected, and incidentally present a problem. Subsequent stages involve strong nuclear interactions. The energy liberated is transmitted principally in the form of X-rays from the core to the photosphere; electromagnetic interactions are involved here and in the transmission of heat and light to the outside universe. Although the strong nuclear reactions proceed rapidly, the overall timescale is set by the very slow first-stage weak interaction, which alone guarantees the long life of the solar system. The main point to be made is that all four fundamental interactions play a vital and balanced role in stellar evolution.

To indicate the relative magnitudes of the four types of interaction, the comparative strengths of the force between two protons when just in contact are very roughly as follow

strong	electromagnetic	weak	gravity	(1.8)
1	$10^{-2}$	$10^{-7}$	$10^{-39}$	

The timescales for the decay of unstable particles via one or other of the fundamental interactions are also very different. As detailed in Table 2.2, a typical mean lifetime  $\tau$  for decay through a weak interaction is  $10^{-10}$  s, which is easily measurable, while that for a strong interaction will be about  $10^{-23}$  s, which cannot be measured directly. However, the Uncertainty Principle relates the lifetime and the uncertainty in energy of a state. An unstable particle does not have a unique mass, but a distribution with ‘width’  $\Gamma = \hbar/\tau$ . So, when  $\tau$  is very short, its value can be inferred from the measured width  $\Gamma$ .

### 1.2.3 Limitations of the Standard Model

Most of the material in this text will be presented in the context of the Standard Model of particle physics. This provides an extremely compact and successful description of the properties of the fundamental constituents discussed above, as well as of the electromagnetic, weak and strong interactions between them. It accounts for an enormous body of experimental data, from laboratory experiments ranging up to the highest available collision energies, of order 1 TeV ( $10^{12}$  electronvolts). However, the Standard Model does have limitations. Gravitational interactions are not included, and persistent attempts over many years to find a way of incorporating gravity have made little progress. In the Standard Model, neutrinos are assumed to be massless, but there is growing evidence, from the solar and atmospheric anomalies discussed in Chapter 9, that neutrinos do have finite masses; and this is regarded as one possible manifestation of physics beyond that encompassed by the Standard Model. The model is also somewhat inelegant, as it contains some 17 arbitrary parameters (masses, mixing angles, coupling constants etc.), and one has to ask where all those numbers come from. The origin of the parameters and the underlying reasons for the ‘xerox copies’ – six quark and six lepton flavours – is not at all understood. As we shall see, it appears that in trying to understand some of the major features of our universe, such as the preponderance of ‘dark matter’ and the large matter–antimatter asymmetry, we will also require new and presently unknown physics beyond that of the Standard Model, as discussed in Chapters 9 and 10. But equally, it seems fairly certain that the model will form an integral and important part of a more complete theory of particles in the far future.

## 1.3 Particle classification: fermions and bosons

Fundamental particles are of two types; particles with half-integral spin ( $\frac{1}{2}\hbar, \frac{3}{2}\hbar, \dots$ ) are called fermions because they obey Fermi–Dirac statistics, while those with integral spin ( $0, \hbar, 2\hbar, \dots$ ) obey Bose–Einstein statistics and are called bosons.

The statistics obeyed by a particle determines how the wavefunction  $\psi$  describing an ensemble of identical particles behaves under interchange of any pair of particles, say 1 and 2. Clearly the probability  $|\psi|^2$  cannot be altered by the interchange  $1 \leftrightarrow 2$ , since the particles are indistinguishable. Thus, under interchange  $\psi \rightarrow \pm\psi$ . There is a fundamental theorem, called the spin-statistics theorem, which is a sacrosanct principle of quantum field theory and according to which the following rule holds:

under exchange of identical bosons  $\psi \rightarrow +\psi$ ;  $\psi$  is symmetric

under exchange of identical fermions  $\psi \rightarrow -\psi$ ;  $\psi$  is antisymmetric

As an application of this rule, suppose one tries to put two identical fermions in the *same* quantum state. Then the wavefunction under interchange of these two identical particles would not change. It is necessarily symmetric under this operation. But this is forbidden by the above rule, according to which the wavefunction *must* change sign. This gives rise to the famous Pauli principle: two or more identical fermions cannot exist in the same quantum state. However, there are no restrictions on the number of bosons in the same quantum state; an example of this is the laser.

One exciting possible extension beyond the Standard Model is the concept of *supersymmetry*, which predicts that, at a high energy scale, of order  $1 \text{ TeV} = 10^{12} \text{ eV}$ , there should be fermion–boson symmetry. Each fermion will have a boson partner and vice versa. The reasons behind this postulate and the experimental limits on possible supersymmetric particle masses are given in Chapter 9.

### 1.4 Particles and antiparticles

Perhaps the two greatest conceptual advances in physics over the last century have been the theory of relativity and the quantum-mechanical description of phenomena on the atomic or subatomic scale. These led Dirac in 1931 to the prediction of *antiparticles*, i.e. objects with the same mass and lifetime as the corresponding particles but with opposite sign of charge and magnetic moment.

Without discussing the full theory of antiparticles here, we can outline the underlying ideas very briefly. The relativistic relation between the total energy  $E$ , momentum  $p$  and rest mass  $m$  of a particle is

$$E^2 = p^2 c^2 + m^2 c^4 \quad (1.9)$$

which for  $pc \ll mc^2$  can be expanded to give the usual expression for the kinetic energy,

$$T = E - mc^2 = mc^2(1 + p^2/m^2c^2)^{1/2} - mc^2 \simeq p^2/2m$$

However, from (1.9) we see that the total energy  $E$  can in principle assume negative as well as positive values,

$$E = \pm \sqrt{p^2 c^2 + m^2 c^4} \quad (1.10)$$

Classically, negative energies for free particles appear to be completely meaningless. In quantum mechanics, however, we represent the amplitude of an infinite stream of particles, say electrons, travelling along the positive  $x$ -axis with

3-momentum  $p$  by the plane wavefunction

$$\psi = Ae^{-i(Et - px)/\hbar} \quad (1.11)$$

where the angular frequency is  $\omega = E/\hbar$ , the wavenumber is  $k = p/\hbar$  and  $A$  is a normalisation constant. As  $t$  increases, the phase advances in the direction of increasing  $x$ . Formally, however, (1.11) can also represent particles of energy  $-E$  and momentum  $-p$  travelling in the negative  $x$ -direction and *backwards in time* (i.e. replacing  $Et$  by  $(-E)(-t)$  and  $px$  by  $(-p)(-x)$ ):

$$\begin{array}{ccc} E > 0 & \text{or} & E < 0 \\ \xrightarrow{\quad} & & \xleftarrow{\quad} \\ t_1 & t_2 & t_1 \quad t_2 \quad (t_2 > t_1) \end{array}$$

Such a stream of negative electrons flowing backwards in time is equivalent to positive charges flowing forward, and thus having  $E > 0$ . Hence, the negative energy particle states are connected with the existence of positive energy antiparticles of exactly equal but opposite electrical charge and magnetic moment, and otherwise identical. The *positron* – the antiparticle of the electron – was discovered experimentally in 1932 in cloud chamber experiments with cosmic rays (see Figure 1.2).

Dirac's original picture of antimatter, developed in the context of electrons, was that the vacuum actually consisted of an infinitely deep sea of completely filled negative energy levels. A positive energy electron was prevented from falling into a negative energy state, with release of energy, by the Pauli principle. If one supplies energy  $E > 2mc^2$ , however, a negative energy electron at  $A$  in Figure 1.3 could be lifted into a positive energy state  $B$ , leaving a 'hole' in the sea corresponding to creation of a positron together with an electron. However, such a picture is not valid for the pair creation of bosons.

In non-relativistic quantum mechanics, the quantity  $\psi$  in (1.11) is interpreted as a single-particle wavefunction, equal to the probability amplitude of finding the particle at some coordinate. In the relativistic case however, multi-particle states are involved (with the creation of particle–antiparticle pairs) and, strictly speaking, the single-particle function loses its meaning. Instead,  $\psi$  has to be treated as an *operator that creates or destroys particles*. Negative energies are simply associated with destruction operators acting on positive energy particles to reduce the energy within the system. The absorption or destruction of a negative energy particle is again interpreted as the creation of a positive energy antiparticle, with opposite charge, and vice versa. This interpretation will be formalised in the discussion of Feynman diagrams in Chapter 2.

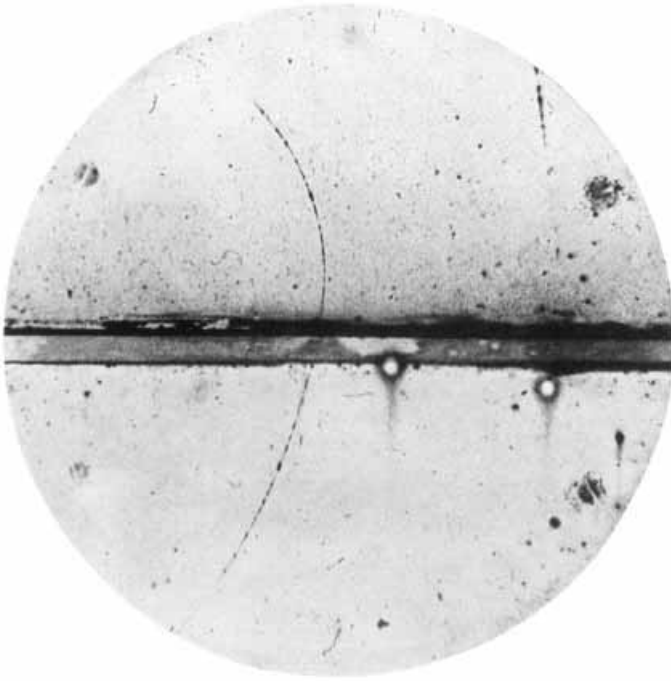


Fig. 1.2. The discovery of antimatter. The picture shows the track of a positron observed by Anderson in 1932 in a cloud chamber placed in a magnetic field and exposed to cosmic rays. Note that the magnetic curvature of the track in the upper half of the chamber is greater than that in the lower half, because of the loss of momentum in traversing the metal plate; hence the particle was proved to be positively charged and travelling upwards. This discovery was confirmed a few months later by Blackett and Occhialini (1933). With a cloud chamber whose expansion was triggered by electronic counters surrounding it (rather than the random expansion method of Anderson) they observed the first examples of the production of  $e^+e^-$  pairs in cosmic ray showers. The antiparticle of the proton – the antiproton – was first observed in accelerator experiments in 1956, but the bound state of positron and antiproton, i.e. the anti-hydrogen-atom, not until 1995.

The existence of antiparticles is a general property of both fermions and bosons, but for fermions only there is a conservation law: the difference in the number of fermions and antifermions is a constant. Formally one can define a fermion number,  $+1$  for a fermion and  $-1$  for an antifermion, and postulate that the total fermion number is conserved. Thus fermions and antifermions can only be created or destroyed in pairs. For example, a  $\gamma$ -ray, in the presence of a nucleus to conserve momentum, can ‘materialise’ into an electron–positron pair and an  $e^+e^-$  bound state, called positronium, can annihilate to two or three  $\gamma$ -rays. An example of an  $e^+e^-$  pair is shown in Figure 1.4.

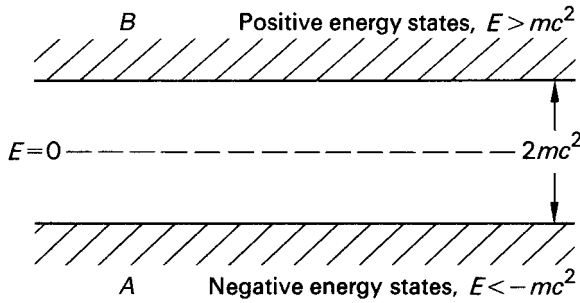


Fig. 1.3. Dirac picture of  $e^+e^-$  pair creation, when an electron at A is lifted into a positive energy state at B, leaving a 'hole' in the negative energy sea, i.e. creating a positron.

### 1.5 Free particle wave equations

The relativistic relation between energy, momentum and mass is given in (1.9):

$$E^2 = p^2 c^2 + m^2 c^4$$

If we replace the quantities  $E$  and  $p$  by the quantum mechanical operators

$$E_{\text{op}} = i\hbar \frac{\partial}{\partial t}, \quad p_{\text{op}} = -i\hbar \nabla = -i\hbar \frac{\partial}{\partial \mathbf{r}} \quad (1.12)$$

where  $\mathbf{r}$  is the position vector, we get the *Klein–Gordon wave equation*

$$\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi - \frac{m^2 c^2}{\hbar^2} \psi \quad (1.13)$$

As described above, it is often more convenient to work in units such that  $\hbar = c = 1$ , in order to avoid writing these symbols repeatedly, so that the above equation becomes

$$\frac{\partial^2 \psi}{\partial t^2} = (\nabla^2 - m^2) \psi \quad (1.14)$$

This wave equation is suitable for describing spinless (or scalar) bosons (since no spin variable has been introduced). In the non-relativistic case, if we define  $E = p^2/(2m)$  as the kinetic energy rather than the total energy then substituting the above operators gives the *Schrödinger wave equation* for non-relativistic spinless particles:

$$\frac{\partial \psi}{\partial t} - \frac{i}{2m} \nabla^2 \psi = 0 \quad (1.15)$$

Note that the Klein–Gordon equation is second order in the derivatives, while

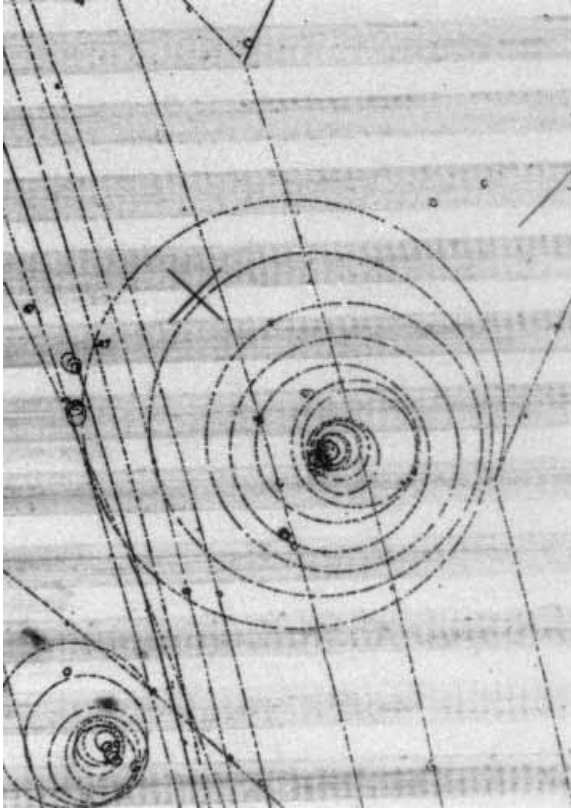


Fig. 1.4. Observation of an electron–positron pair in a bubble chamber filled with liquid hydrogen. An incoming negative pion – itself a quark–antiquark combination – undergoes charge exchange in the reaction  $\pi^- + p \rightarrow n + \pi^0$ . This strong interaction is followed by electromagnetic decay of the neutral pion. The usual decay mode is  $\pi^0 \rightarrow 2\gamma$ , the  $\gamma$ -rays then converting to  $e^+e^-$  pairs in traversing the liquid. In about 1% of events, however, the decay mode is  $\pi^0 \rightarrow \gamma e^+e^-$ : the second  $\gamma$ -ray is ‘internally converted’ to a pair. Since the neutral pion lifetime is only  $10^{-16}$  s, the pair appears to point straight at the interaction vertex. The bubble chamber detector was invented by Glaser in 1952. It consists basically of a tank of superheated liquid (hydrogen in the above example), prevented from boiling by application of an overpressure. When the overpressure is released, boiling initially occurs along the trails of charged ions left behind by passage of fast charged particles through the liquid and leaves tracks of bubbles that can be photographed through a front window. As in the cloud chamber, a magnetic field normal to the plane of the picture serves to measure particle momentum from curvature.

the Schrödinger equation is first order in time and second order in space. This is unsatisfactory when we are dealing with high energy particles, where the description of physical processes must be relativistically invariant, with space and time coordinates occurring to the same power.



Dirac set out to formulate a wave equation symmetric in space and time, which was *first order in both derivatives*. The simplest form that can be written down is that for massless particles, in the form of the *Weyl equations*

$$\frac{\partial \psi}{\partial t} = \pm \left( \sigma_1 \frac{\partial \psi}{\partial x} + \sigma_2 \frac{\partial \psi}{\partial y} + \sigma_3 \frac{\partial \psi}{\partial z} \right) = \pm \boldsymbol{\sigma} \cdot \frac{\partial}{\partial \mathbf{r}} \psi \quad (1.16)$$

Here the  $\sigma$ 's are unknown constants. In order to satisfy the Klein–Gordon equation (1.14), we square (1.16) and equate coefficients, whence we find

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$$

$$\sigma_1 \sigma_2 + \sigma_2 \sigma_1 = 0, \quad \text{etc.} \quad (1.17)$$

$$m = 0$$

These results hold for either sign on the right-hand side of (1.16), and both must be considered. The  $\sigma$ 's cannot be numbers since they do not commute, but they can be represented by matrices, in fact the equations (1.17) define the  $2 \times 2$  Pauli matrices, which we know from atomic physics to be associated with the description of the spin quantum number of the electron:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (1.18)$$

Using (1.12) we can also express (1.16) in the forms

$$E\chi = -\boldsymbol{\sigma} \cdot \mathbf{p}\chi \quad (1.19a)$$

$$E\phi = +\boldsymbol{\sigma} \cdot \mathbf{p}\phi \quad (1.19b)$$

where  $E$  and  $\mathbf{p}$  are the energy and momentum operators.  $\chi$  and  $\phi$  are two-component wavefunctions, called spinors, and are separate solutions of the two Weyl equations, and  $\boldsymbol{\sigma}$  denotes the Pauli spin vector, with Cartesian components  $\sigma_1, \sigma_2, \sigma_3$  as above. As indicated below, the two Weyl equations have in total four solutions, corresponding to particle and antiparticle states with two spin substates of each.

If the fermion mass is now included, we need to enlarge (1.16) or (1.19) by including a mass term, giving the *Dirac equation*,

$$E\psi = (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m)\psi \quad (1.20a)$$

Here, the matrices  $\boldsymbol{\alpha}$  and  $\beta$  are  $4 \times 4$  matrices, operating on four-component

(spinor) wavefunctions (particle, antiparticle and two spin substates for each). The matrices  $\alpha$  and  $\beta$  are

$$\alpha = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where each element denotes a  $2 \times 2$  matrix and ‘1’ denotes the unit  $2 \times 2$  matrix. The matrix  $\alpha$  has three components, just as does  $\boldsymbol{\sigma}$  in (1.18). Here, we have quoted the so-called Dirac–Pauli representation of these matrices, but other representations are possible.

Usually, the Dirac equation is quoted in a covariant form, using (1.12) in (1.20a), as

$$\left( i\gamma_\mu \frac{\partial}{\partial x_\mu} - m \right) \psi = 0 \quad (1.20b)$$

where the  $\gamma_\mu$  (with  $\mu = 1, 2, 3, 4$ ) are  $4 \times 4$  matrices related to those above. In fact

$$\gamma_k = \beta\alpha_k = \begin{pmatrix} 0 & \boldsymbol{\sigma}_k \\ -\boldsymbol{\sigma}_k & 0 \end{pmatrix}, \quad k = 1, 2, 3 \quad \text{and} \quad \gamma_4 = \beta \quad (1.20c)$$

The Dirac equation is fully discussed in books on relativistic quantum mechanics, and we have mentioned it here merely for completeness; we shall not discuss it in detail in this text. Occasionally we shall need to quote results from the Dirac equation without derivation. However, it turns out that, in most of the applications with which we shall be dealing in high energy physics, the fermions have extreme relativistic velocities so that the masses can be neglected and the Dirac equation breaks down into the two much simpler, decoupled, Weyl equations as described above.

## 1.6 Helicity states: helicity conservation

For a massless fermion of positive energy,  $E = |\mathbf{p}|$  so that (1.19a) satisfies

$$\frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{|\mathbf{p}|} \chi = -\chi \quad (1.21)$$

The quantity

$$H = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{|\mathbf{p}|} = -1 \quad (1.22)$$

is called the *helicity* (or handedness). It measures the sign of the component of spin of the particle,  $j_z = \pm \frac{1}{2}\hbar$ , in the direction of motion ( $z$ -direction). The  $z$ -component of spin and the momentum vector  $\mathbf{p}$  together define a screw sense, as in Figure 1.5.  $H = +1$  corresponds to a right-handed (RH) screw, while particles with  $H = -1$  are left-handed (LH).

The solution  $\chi$  of (1.19a) represents a LH, positive energy, particle but it can also represent a particle with negative energy  $-E$  and momentum  $-\mathbf{p}$ . Thus  $-E\chi = -\boldsymbol{\sigma} \cdot (-\mathbf{p})\chi$ , or  $H = \boldsymbol{\sigma} \cdot (-\mathbf{p})/|\mathbf{p}| = +1$ . This state is interpreted, as before, as that of the antiparticle. Thus, (1.19a) represents *either* a LH particle *or* a RH antiparticle, while the independent solution (1.19b) corresponds to a RH particle or a LH antiparticle state.

Helicity is a well-defined, Lorentz-invariant quantity for a massless particle, for the simple reason that such a particle travels at velocity  $c$ . In making a Lorentz transformation to another reference frame of relative velocity  $v < c$ , it is therefore impossible to reverse the helicity. As discussed below, neutrinos have very small, possibly even zero, masses, and are well described by one of the two Weyl equations. By contrast, it turns out that solutions of the Dirac equation (1.20), with its finite mass term, are *not* pure helicity eigenstates but some admixture of LH and RH functions. However, provided they are extreme relativistic, massive fermions (electrons for example) can also be described well enough by the Weyl equations.

For interactions involving *vector* or *axial vector fields*, i.e. those mediated by vector or axial vector bosons, *helicity is conserved in the relativistic limit*. The reason is that such interactions do not mix the separate LH and RH solutions of the Weyl equations. This means for example that a LH lepton, undergoing scattering in such an interaction, will emerge as a LH particle, irrespective of the angle of scatter, provided it is extreme relativistic. On the other hand, a *scalar* interaction does not preserve the helicity and does mix LH and RH states. In the Dirac equation, the mass term represents such a scalar-type interaction and because of its presence, massive leptons with  $v$  less than  $c$  are superpositions of LH and RH helicity states. In the successful theory of electroweak interactions discussed in Chapter 8, the elementary leptons and bosons start out as massless particles. Scalar field particles, called Higgs bosons, are associated with an all-pervading scalar field which is postulated to interact with, and give mass to, these hitherto massless objects.

Helicity conservation holds good in the relativistic limit for any interaction that has the Lorentz transformation properties of a vector or axial vector, and it therefore applies to strong, weak and electromagnetic interactions, which are all mediated by vector or axial vector bosons. Consequently, in a scattering process at high energy, e.g. of a quark by a quark or a lepton by a quark or lepton, a LH particle remains LH, and a RH particle remains RH. This fact, together with the conservation of angular momentum, determines angular distributions in many interactions, as described later in the text.

### 1.7 Lepton flavours

The masses, or mass limits, of the known leptons are given in Table 1.4. The masses are quoted in energy units, i.e. the value of the rest energy  $mc^2$ , in eV