Vortex Methods

The goal of this book is to present and analyze vortex methods as a tool for the direct numerical simulation of incompressible viscous flows.

Vortex methods have matured in recent years, offering an interesting alternative to finite-difference and spectral methods for high-resolution numerical solutions of the Navier–Stokes equations. In the past two decades research in the numerical analysis aspects of vortex methods has provided a solid mathematical background for understanding the convergence features of the method and several new tools have been developed to generalize its application. At the same time vortex methods retain their appealing physical character that was the motivation for their introduction.

Scientists working in the areas of numerical analysis and fluid mechanics will benefit from this book, which may serve both communities as both a reference monograph and a textbook for computational fluid dynamics courses.

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Vortex Methods:
Theory and Practice

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Preface

The goal of this book is to present and analyze vortex methods as a tool for the direct numerical simulation of incompressible viscous flows. Its intended audience is scientists working in the areas of numerical analysis and fluid mechanics. Our hope is that this book may serve both communities as a reference monograph and as a textbook in a course of computational fluid dynamics in the schools of applied mathematics and engineering.

Vortex methods are based on the discretization of the vorticity field and the Lagrangian description of the governing equations that, when solved, determine the evolution of the computational elements. Classical vortex methods enjoy advantages such as the use of computational elements only in cases in which the vorticity field is nonzero, the automatic adaptivity of the computational elements, and the rigorous treatment of boundary conditions at infinity. Until recently, disadvantages such as the computational cost and the inability to treat accurately viscous effects had limited their application to modeling the evolution of the vorticity field of unsteady high Reynolds number flows with a few tens to a few thousands computational elements. These difficulties have been overcome with the advent of fast summation algorithms that have optimized the computational cost and recent developments in numerical analysis that allow for the accurate treatment of viscous effects. Vortex methods have reached today a level of maturity, offering an interesting alternative to finite-difference and spectral methods for high-resolution numerical solutions of the Navier–Stokes equations. In the past two decades research in numerical analysis aspects of vortex methods has provided a solid mathematical background for understanding the accuracy and the stability of the method. At the same time vortex methods retain their appealing physical character that, we believe, was the motivation for their introduction.

Historically, simulations with vortex methods date back to the 1930s, with Rosenhead's calculations by hand of the Kelvin–Helmholtz instabilities. For several decades the grid-free character and the physical attributes of vortex methods were exploited in the simulation of unsteady separated flows. Simultaneously, the close relative of vortex methods, the surface singularity (panel) methods were developed and still remain as a powerful engineering tool for the prediction of loads in aerodynamic configurations. The modern developments of vortex methods originate in the works of Chorin in the 1970s (in particular for the design of random-walk methods), and in the three-dimensional calculations of Leonard (in the USA) and Rehbach (in France). These numerical works soon motivated the interest of applied mathematicians for understanding the convergence properties of these methods in the early 1980s. The very first complete convergence analysis was done in the USA by Hald, followed by Beale and Majda. In Europe, at about the same time, the group of Raviart undertook this research in parallel with the analysis of particle methods for plasma physics.

The past two decades have seen significant developments in the design of fast multiple methods for the efficient evaluation of the velocity field by Greengard and Rohklin; design and numerical analysis of new accurate methods for the treatment of viscous effects (in the group of Raviart); a number of benchmark applications demonstrating the capabilities of vortex methods for Direct Numerical Simulations of unsteady separated flows in Leonard's group at Caltech; and finally a deeper understanding of convergence properties, with convergence proofs of random-walk methods by Long and Goodman, and convergence proof for point vortex methods by Hou and co-workers. In this book we discuss these recent developments by mixing as much as possible the points of view of numerical analysis and fluid mechanics. We indeed believe that a remarkable feature of vortex methods is that, unlike other numerical methods, such as finite differences and finite elements, they are fundamentally linked to the physics they aim to reproduce.

Concerning the numerical analysis in the inviscid case, several approaches are now available since the pioneering work of Hald. In this book we focus on a convergence proof based on the tools developed approximately 10 years ago around the concept of weak solutions to advection equations in distribution spaces. There are three reasons that motivated this choice: the notion of weak measure solution is the central mathematical concept in particle methods; second, within this framework the convergence analysis is inherently linked to the structure of the equations, and it applies in many apparently different situations: two- and three-dimensional grid-free methods, including vortex filament methods, and vortex-in-cell methods. Convergence properties of contour dynamics methods, which are not explicitly covered in this book, are also easily understood with these tools. Finally, we believe that the present convergence proof gives optimal results, in particular with respect to the smoothness of the flow, leading to error estimates similar to those of more traditional numerical methods. In Chapter 2 we present this convergence theory for two-dimensional inviscid flows.

Throughout the book, we have tried to maintain a balance between plain numerical analysis and a more qualitative description of the methods. We have given particular attention to conservation properties that are essential in the design of vortex methods. For instance, the energy conservation in two-dimensional schemes, which follows from the Hamiltonian character of the particle motion, is a feature that distinguishes vortex methods from Eulerian schemes. For three-dimensional schemes, covered in Chapter 3, the conservation of circulation has long been an argument in favor of vortex filament methods against the vortex particle methods. We discuss several ways now available to enforce conservation in this second class of methods. We also address practical issues related to the constraint of divergence-free vorticity fields when using vortex particles in three dimensions.

In Chapter 4 we discuss boundary conditions for inviscid flow vortex simulations. We present this in a formal way by considering the Poincaré identity that basically provides the kinematic boundary conditions (no-through-flow) for the Biot–Savart law for flows around solid boundaries. We discuss the method of surface singularities and panel methods as special types of vortex methods. The incorporation of these techniques along with vortex shedding models and the Kutta condition in engineering calculations is discussed.

For viscous flows, besides the popular random-walk method, we have emphasized in Chapter 5 the so-called deterministic vortex methods. These schemes, started at Ecole Polytechnique in 1983, have now given rise to several variants. Applications for two- and three-dimensional flows demonstrate the practicality of viscous schemes and demonstrate the ability of vortex methods to simulate viscous effects accurately, while maintaining the Lagrangian character of the method. Concerning the numerical analysis, we postulate that convergence for the Navier–Stokes equations can be understood in the light of convergence for the Euler equations and for linear convection–diffusion equations. This approach is somewhat biased, as the technical difficulties in the numerical analysis for the full Navier–Stokes equations are much more than the sum of difficulties for the Euler and linear equations, but it makes the presentation more cohesive. We thus focused our attention on linear convection–diffusion equations.

In Chapter 6 we discuss viscous vortex methods for flows evolving in a domain containing solid boundaries. Here the proof of convergence is a far less easy task. It is possible to carry out a numerical analysis, but at the cost of doing constructions (like extending the vorticity support outside the domain) that are not possible for practical applications. We have thus preferred to stress here the difficulties and indicate various attempts to overcome them. To our knowledge there is no completely satisfactory solution for general geometries, in particular because of the need to regularize vortices near the boundary. This problem, already present for inviscid flows, is even more crucial for viscous flows since one has to design, and to implement in the context of a viscous scheme, vorticity boundary conditions. In that context we discuss the no-slip boundary condition and its equivalence with the vorticity boundary condition. We emphasize the case of Neumann-type conditions and investigate their links with classical vorticity generation algorithms. Integral techniques for the implementation of these boundary conditions are then presented and illustrated by applications of vortex methods in the direct numerical simulation of bluff body flows.

In Chapter 7 we discuss the issue of particle distortion inherent in all Lagrangian methods. We argue the necessity of maintaining a somewhat regular Lagrangian grid, from the point of view of numerical analysis, as well as of its practical ramifications. We present methodologies to achieve this goal either by manipulating the particle locations or by processing the circulation of the particles.

In Chapters 4 and 6 we stress the difficulties of vortex methods in dealing with bounded flows. We are indeed convinced that one can get most of the power of vortex methods by combining them, in what we would call hybrid schemes, with Eulerian methods that precisely may avoid difficulties inherent in particle methods near boundaries. A broad class of hybrid schemes, including domain decomposition techniques, is described and illustrated in Chapter 8.

Chapter 1 is an introduction to the notation and the main properties of incompressible fluids. Finally, in the appendices we have included some key concepts of numerical analysis that would help make this book self contained. We have also included a description of what we consider the muscle of vortex methods, the fast summation technique.

As a final thought, we stress that in this book we do not attempt a thorough review of the progress in vortex methods and their applications in the past decades. In particular, applications to the important field of reacting and compressible flows or free surface flows are not explicitly covered. For this we refer to the proceedings [9, 10, 11, 22, 39, 87] of the workshops that have been devoted since 1987 to vortex methods and, more generally, vortex dynamics. We hope, however, that the book demonstrates some important recent advances in these methods and helps make them recognized as a valuable tool in computational fluid dynamics.

Grenoble,
Zürich,
December 1998.

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