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Rafael Jose Iorio and Valeria de Magalhaes Iorio
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Fourier Analysis and Partial Differential Equations

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Dedicated to the memory of Tosio Kato

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Preface

This book is the outcome of several courses and seminar talks held at the Instituto de Matemática Pura e Aplicada (IMPA) over the years. It is a greatly modified version of a previous work by the authors, *Equações Diferenciais Parciais, Uma Introdução*, (Projeto Euclides, IMPA, 1978). It has a twofold purpose, namely to introduce the student to the basic concepts of Fourier analysis and provide illustrations of recent applications where these concepts were used to study various properties of the solutions of some important nonlinear evolution equations.

The text is divided into three parts. The first one, containing Chapters 1 to 3, deals with Fourier series and periodic distributions. Chapters 4 to 6 belong to the second part, which contains applications of Fourier series and periodic distributions to partial differential equations. Chapters 7 and 8, in the third part, are more advanced and deal with some nonperiodic problems.

Chapter 1 presents some very classical material on PDEs, such as classification into types, separation of variables and maximum principles for the heat and Laplace equations. It is by no means a comprehensive account of such topics. Rather, it's purpose is to establish the basic language used throughout the work and to provide a collection of definitions and results needed in the remainder of the book. The following two chapters deal with Fourier series and some of its applications, first in a classical setting and then in the scenario provided by $\mathscr{P}'$, the space of periodic distributions. We include some general topological concepts that will be needed later on, and introduce L^2-type periodic Sobolev spaces using the fact that the Fourier transform is an isomorphism from $\mathscr{P}'$ onto the collection of all complex sequences of slow growth. In this way we reduce all our considerations to spaces of sequences and thus avoid the use of the Lebesgue integral until very late in the game. Chapter 4

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concentrates on applications of the theory developed in the preceding chapters to linear evolution equations. We study a large number of such objects, including the heat, (free) Schrödinger and the wave equations. Although the chapter is interesting in its own right, its main purpose is to lay the groundwork for the applications to nonlinear evolution equations given in Chapters 5 and 6. We have included, for completeness' sake, a section summarizing the basic theory of semigroups of operators. Its purpose is to provide an abstract point of view for our treatment of linear evolution equations. It can be skipped without consequence to the understanding of the remainder of the text. Part Three addresses some situations that do not occur in the periodic setting, such as well-posedness in weighted Sobolev spaces and problems with initial conditions with 'infinite mass', that is, initial data that does not belong to $L^2(\mathbb{R})$. This is done in Chapter 8. Chapter 7 discusses the basic concepts of the theory of distributions, Sobolev spaces and presents applications to linear evolution equations, with emphasis on the heat and Schrödinger equations. Here we lay the groundwork for the applications studied in the final chapter. There are two appendixes. The first one summarizes the ODE theory used in the text while the second describes some technical commutator estimates needed to deal with the Korteweg–de Vries and related equations.

As is almost always the case, the choice of the topics discussed in this book is a direct consequence of the tastes and research interest of the authors. We have refrained from the study of classical elliptic theory, since there are many excellent works on the subject, and decided to concentrate on linear and nonlinear evolutions equations. As mentioned above, Lebegue's theory of integration is needed only very late in the book. The first point where it has to be used is in the proof of local-well-posedness of the Korteweg–de Vries equation presented in section 3 of Chapter 6. At that point we need Pettis' theorem on weakly measurable functions and the concept of absolutely continuous functions defined on an interval with values in a Banach space. However, the reader who is unfamiliar with these ideas may skip the section, because all the relevant results are stated in Sections 1 and 2. In Part Three, it is no longer feasible to avoid the theory of integration, and there we assume that the reader is familiar with the essentials of the theory as presented in the books by Bartle, Royden or Rudin, mentioned in the bibliography. We emphasize that our avoidance of using Lebesgue integration is intended to make most of the book available to advanced undergraduates or beginning graduates that are still unfamiliar with the theory. In fact, familiarity

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with integration theory would enhance the understanding of the book, and make it more pleasurable to read.

A final word about prerequisites. We assume that the reader is familiar with the material usually covered in functional analysis courses, up to the theory of compact operators. We also assume familiarity with the basic theory of ordinary differential equations, more specifically with the results presented in Appendix B.

Finally the authors wish to thank our friends Carlos Augusto Isnard (IMPA), Felipe Linares (IMPA) and Marcia Scialom (IMECC\UNICAMP) for several interesting conversations on the subject matter of this book and for reading various parts of the original manuscript. It goes without saying that any mistakes found in the text are the authors', and only the authors', responsibility. And last, but not least, our thanks to the long suffering and patient David Tranah, of Cambridge University Press, who gave us all the support we needed while writing this book.

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