CAMBRIDGE

Cambridge University Press
978-0-521-62104-5 - Quickest Detection
H. Vincent Poor and Olympia Hadjiliadis
Index
More information

Index

Note: footnotes are indicated by suffix 'n'

ϵ-contaminated classes for robust hypothesis testing 198–9
σ-algebra 6
σ-field 6

adaptive quickest detection, change detection in case of uncertainty 200–1
almost-sure convergence 16, 17
  non-negative martingales/supermartingales 21, 55
American call options 46–9
asymptotically uniformly most powerful (AUMP) test 207
autoregressive (AR) process 202n6
autoregressive moving average (ARMA) processes 204
average cost of errors 67
average sample number (asn)
  sequential probability ratio test 80, 96
    calculation of 98
  SPRT-like stopping time, calculation of 208

backward induction 42–3
Bayesian optimal sequential decision rule 73
Bayesian quickest detection
  Brownian observations 109–15
  decentralized version 184–9
  game theoretic formulation 125–8
  Poisson observations 115–22
  probability maximizing approach 122–4
  Shiryaev's problem
    continuous-time case 109–22
    discrete-time case 103–9
Bellman equation 182, 192
Bernoulli random variables 108
Bernoulli trials, sequential probability ratio test 76, 77
Borel–Cantelli lemmas 18
Borel σ-field 7n, 14n
bounded convergence theorem 17
  applications 55, 89, 104, 112, 114, 120, 147
Brownian motion 28–30
  basic properties 29–30
  sample-path continuity 29, 89, 113
  with two-sided alternatives 167–71
  with unknown drift parameter, non-Bayesian quickest detection for 152–4
Brownian motion models
  with distinct drift parameters 81–6
  with distinct variance parameters 86
Brownian observations
  Bayesian quickest detection 109–15
  non-Bayesian quickest detection 144–50
  sequential detection 81–90

càdlàg functions 26
  space of 93, 115, 154
càdlàg martingale 38
càdlàg process 31, 117
Cameron–Martin formula 38, 82
  applications 82, 83, 94
cardiac patients, monitoring of 1
central limit theorem 18
  applications 209
change detection, applications covered 1
change-point detection 1
  see also Bayesian quickest detection; non-Bayesian quickest detection
characteristic function 10
Chebychev's central limit theorem 18
classical sequential detection problem, as optimal stopping problem 65
climate modeling applications 1
coin-flipping experiment 168
computer network monitoring 1
condition monitoring 130
conditional expectation 11–13
  in examples 14–15, 23
  properties 23, 186
continuity properties 60, 61
continuous random variables 8
continuous-time Markov process 60
continuous-time semimartingales 32–4
  with stationary and independent increments and integrable processes 28
convergence in distribution 16, 17

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-62104-5 - Quickest Detection
H. Vincent Poor and Olympia Hadjiliadis
Index
More information

convergence in probability 16, 17
  relationship with L^p convergence 17
counter-terrorism applications 1
cumulative probability distribution function (cdf) 8
cumulative sum (CUSUM) test 3, 128, 132
CUSUM process(es) 145, 164
  log 152, 154
  normalized 169
  optimality 203
  threshold and 155, 156
CUSUM stopping time(s) 132, 139, 140, 144, 145–6, 150, 152, 154, 155
  asymptotic optimality 157, 158–61, 168–71, 212
  classical 2-CUSUM stopping time 168, 171
    normalized 169
  connection with Shiryaev–Roberts–Pollack stopping time 163
  as equalizer rule 134, 143, 146, 155
  generalized 200
  modified drift 2-CUSUM stopping time 168, 171
    definition(s) 169
  normalized 169
  one-sided 153, 154, 157
  optimality 148–50, 152, 155, 195
  and sequential probability ratio test 159–61, 201
  worst-case detection delay for 145
cycle (in Ritov's game-theoretic problem) 127

decentralized quickest detection, with fusion center 184–9
decentralized sequential detection
  advantages and disadvantages 174
  with fusion center 176–84
  without fusion 189–94
decentralized system
  configurations 175
  information flow in 175–6, 177, 184
  quantized information in 174, 175, 176
delay differential equation (DDE) 98
  solution(s) 99, 155, 156
detection delay
  mean value used (in Bayesian quickest detection) 3, 103
  worst-case value used (in non-Bayesian quickest detection) 4, 130, 131, 143
detection-delay/false-alarm rate trade-off, optimization of 3, 102, 103, 131, 165
detection-delay penalties
  in Bayesian quickest detection 124–5
  in non-Bayesian quickest detection 157, 162
diffusion approximations 86
diffusion processes *see* Brownian motion; Itô processes
discrete random variables 8
disorder detection 1
dominated convergence theorem 17
  applications 105, 114, 187
Doob decomposition theorem 20
  applications 51, 53
  increasing process in 20, 23
Doob–Meyer decomposition 27
doubly stochastic Poisson process 31
drift parameter (Brownian motion) 28, 81
dynamic programming 46, 177

econometrics applications 1
efficient score statistic 203
environmental applications 1
equalizer rule, CUSUM stopping time as 134, 143, 146, 155
error probabilities
  bounds 77–8
  relationship with expected runlengths 75–6
  relationship with thresholds 74–5, 85
errors
  average cost of 67
  cost of, tradeoff against sampling cost 66–72
errors cost/sampling cost optimization problem 67–72
  example 72–3
  as Markov optimal stopping problem 67, 69, 70
essential supremum (esssup) 42n, 131n1
event class 6
events 7
excessive functions 60
excessive majorants 60–1
expected runlengths 78
  relationship with error probabilities 75–6
expected value(s) 8–10
expiration date (in American call options) 46–7
exponential detection-delay penalty 125, 131
exponential family 205
extended random variable(s) 8, 11
  as limits of monotonically increasing sequences 53

failure times, model for 106
fair sequence 19
false-alarm rate
  in Bayesian quickest detection 3, 102, 103, 125, 165
  in non-Bayesian quickest detection 131, 144, 145, 164, 165
    constraint 131, 171–2
Fatou's lemma 17
  applications 57, 105, 186
filtration 19, 20
  continuous-time analog 27
  example 20
finance applications 1
finite variation process 32–3
Fisher information matrix 203
fixed-sample testing 167
  compared with sequential testing 65
fraud detection 1

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-62104-5 - Quickest Detection
H. Vincent Poor and Olympia Hadjiliadis
Index
More information

fusion center (in decentralized system) 174, 175
  decision made by 174, 176
  optimal sequential decision rules 177, 183, 184

generalized Radon–Nikodym derivative 11
geometric prior distribution (of change point) 3, 106
Girsanov's theorem 38, 145

historical texts, analysis of 1
homogeneous Markov processes 41, 60, 69
homogeneous Poisson counting process (HPCP) 31
  in Bayesian quickest detection 115, 116
  in sequential detection 93
homogeneous Poisson processes 31, 116, 154

image analysis applications 1
independence (statistical) 13–14, 17–18
  example 15
independent increments 28
independent likelihood ratio sequences, quickest detection with 201–3
infinitely divisible distributions 30
inhomogeneous Poisson process 31
innovation process 87, 114
innovation theorem 37
  applications 112
inspection systems, change detection in 130
integrable random variable 9
Itô processes 36
  non-Bayesian quickest detection for 150–2
  properties 36–8
  sequential detection for 91–3
Itô's isometry 35
Itô's rule 36–7
  applications 82, 87, 114, 147, 170
  generalized 37, 155
Itô stochastic differential equation 36–7
  applications 82, 87, 114

Jensen's inequality 10
  applications 22, 23, 75, 76, 93, 115

Kolmogorov's inequality 20
  compared with Markov's inequality 20
  continuous-time analog 27
Kolmogorov's strong law of large numbers 18
Kullback–Leibler divergence 131, 151, 196, 207

Lebesgue decomposition theorem 10, 11
Lebesgue–Stieltjes integral 9, 34
likelihood ratio
  under alternative hypothesis 21–2
  under null hypothesis 21
likelihood ratio sequence(s)
  independence 202
  quickest detection with independent 201–3
  sequential detection and 72, 100
linear-time (false detection) penalty 125
local martingales 32–3
locally asymptotically normal (LAN) distributions 203–5
  characteristics 208
  meaning of term 204
Lorden's criterion 144, 157, 165
Lorden's delay penalty 165, 167, 195
Lorden's problem
  continuous-time case 144–57
  discrete-time case 130–41
L^p convergence 16, 17
  relationship with convergence in probability 17

Markov optimal stopping problem(s) 40–1, 46–50, 69, 70
  continuous-time 60–1, 120
  meaning of term 41
Markov optimal stopping theory 2, 40–61
  applications 132, 183
Markov process 40–1, 46, 60
  transition properties 41, 60
Markov's inequality, compared with Kolmogorov's inequality 20
Markov time, meaning of term 24
martingale convergence theorem 20, 27
martingales 19–21
  continuous-time analogs 26–7
  examples 21–3
  gambling analogy 19
measurable space 7
medical diagnosis applications 1
min–max criterion, as delay penalty 162
min–max optimization problem 163
minimal Bayes cost, determination of 90
minimal cost function 71
minimal runlength 73
minimal supermartingale 52
moment-generating function 10
  in examples 22
monotone convergence theorem 17
  applications 53, 56, 57, 89, 111, 117, 133
monotone likelihood ratio quantizer (MLRQ) 189
monotone pointwise limit 71
Moustakides' proof of optimality of Page's CUSUM test 132

navigation systems monitoring 1
network security applications 1
neuroscience applications 1
non-Bayesian quickest detection
  asymptotic results 157–71
    Brownian motion with two-sided alternatives 167–71
    Lorden's approach 158–67
  Brownian motion with unknown drift parameter 152–4

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-62104-5 - Quickest Detection
H. Vincent Poor and Olympia Hadjiliadis
Index
More information

non-Bayesian quickest detection (*cont.*)
  Brownian observations 144–50
  Itô processes 150–2
  Lorden's problem
    continuous-time case 144–57
    discrete-time case 130–41
  Poisson observations 154–7
Novikov condition 38n, 110, 145

odds ratio process 118, 163, 164
operating characteristic (of sequential probability ratio test) 80, 96, 97
optimal stopping problem 40
  detection-delay/false-alarm rate trade-off optimization treated as 103–4, 134
  finite-horizon case
    general case 41–5
    Markov case 46–50
    martingale interpretation 51–2
  infinite-horizon case 50–60
    for bounded reward 52–5
    general case 55–9
    with Markov rewards 59–60
optimal stopping time(s) 45, 56
  in Bayesian quickest detection 106, 107, 109, 121, 122, 124, 127
  in decentralized quickest detection with fusion center 189
  in decentralized sequential detection without fusion 192
  in non-Bayesian quickest detection 134, 138, 165, 168
  in sequential detection 70
option trading, example 46–9
optional sampling theorem 24–5
  applications 54, 55, 105, 112, 187
  continuous-time 27, 88
optional time 27
overview of book 2–3

Page's CUSUM test 3, 128, 132, 140, 142–4
payoff (in optimal stopping problem) 40, 49
penalty functions 103, 124–5
performance analysis 74–81
person-by-person optimal decision rule 179, 187
Poisson observations
  Bayesian quickest detection 115–22
  non-Bayesian quickest detection 154–7
  sequential detection 93–101
Poisson processes 30–2
Pollack's criterion 163, 165
polynomial-type delay penalty 124
pre-probability space 7
prior distribution (of change point) 113, 130
probability, convergence in 16, 17
probability density function (pdf) 9
probability distribution *see* probability measure
probability mass function (pmf) 9
probability measure 6
  properties 7
probability spaces 6–7
probability theory 6–39
product measure 15
public health applications 1

quadratic variation process 23, 33
quickest detection
  applications 1
  Bayesian framework 102–29
  local hypothesis approach 210–12
  non-Bayesian framework 130–73
  with dependent observations 201–12
  with independent likelihood ratio sequences 201–3
  with modelling uncertainty 194–201
    adaptive quickest detection used 200–1
    robust quickest detection used 194–9
quickest detection problem(s) 1, 102
  Lorden's formulation
    continuous-time case 144–57
    discrete-time case 130–41
    Page's test for 132, 140
  Shiryaev's formulation
    continuous-time case 109–22
    discrete-time case 103–9

radio monitoring applications 1
Radon–Nikodym derivatives 10–11
  applications 91, 151, 163
  example 11
  generalized 11
Radon–Nikodym theorem 10
  applications 68, 82, 83, 94
  conditional expectation as corollary to 12
random experiment 6
random sequences 15–18
random variable(s) 7–8
  central moments 10
  expected value(s) 8–10
  limits of sequences 18
  mean of 10
  moments 10
  types 8
  variance of 10
random walk from interval, first exit time 78, 143
regular martingales 23
regular stopping times 25
remote sensing applications 1
renewal theory 4, 142
Riemann–Stieltjes integral 9
Ritov's game-theoretic quickest detection problem 125–8
  solution by Page's test 3, 128, 132, 140

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-62104-5 - Quickest Detection
H. Vincent Poor and Olympia Hadjiliadis
Index
More information

Ritov's proof of optimality of Page's CUSUM test 132
robust quickest detection, change detection in case of uncertainty 194–9
runlengths (of sequential probability ratio tests), relationship with error probabilities 75, 85

sample-path continuity
  Brownian motion 84, 89, 113
  Itô processes 36, 38
  local martingale 33, 34
sample space 6
sampling cost 67
  tradeoff against cost of errors 66–72
security applications 1
seismology applications 1
selection problem, example 43–5
self-exciting Poisson processes 31
semimartingales 19
sequential decision rule (s.d.r.) 66
  applications 67, 72, 82, 84, 90, 92, 94, 177
  for errors cost/sampling cost optimization problem 67, 72
sequential detection
  continuous-time case
    Brownian case 81–90
    compound Poisson case 100–1
    for Itô processes 91–3
    Poisson case 93–100
  local hypothesis approach 205–10
  optimal detection 65–74
  performance analysis 74–81
sequential probability ratio test (SPRT) 3, 73
  average sample number (asn) 80, 96
    calculation of 98
  continuous-time case 84, 91, 92
  and CUSUM 159–61, 201
  operating characteristic (oc) 80, 96, 97
  relationship between performance indices 74–6
  the case of Iro processes 92
  Wald–Wolfowitz property 91
sequential testing, advantages over fixed sample testing 65
Shiryaev–Roberts–Pollack stopping time 162–3, 171, 172, 173
  connection with CUSUM stopping time and Shiryaev's stopping time 163
Shiryaev–Roberts stopping time 162, 163, 172
  optimality 164–5
Shiryaev's criterion 103, 125, 165
Shiryaev's problem
  continuous-time case 109–22
    Brownian observations 109–15
    Poisson observations 115–22
  discrete-time case 103–9
  min–max solution 162
Shiryaev's stopping time 163
simple random variable 8
Snell envelope 2, 137
SPRT *see* sequential probability ratio test
SPRT-like stopping time 206, 208
square-integrable martingales 23, 33
standard Brownian motion 28–9, 37, 38
  applications
    quickest detection 109, 110, 143, 152, 211
    sequential detection 81, 87, 92
stationary increments 28
stationary Markov optimal stopping problem 41
statistical change detection 1
'statistician vs nature' game 125–8
Stefan problem 94
stochastic convergence 16–17
stochastic differential equations 82, 87, 114
stochastic integral 34–9
stochastic optimization problem 185, 195
stochastic processes 15–16
stochastic sequence 40
  Markov representation 41
stopping times 24–6
  continuous-time analogs 27
  localizing sequence 32
  meaning of term 24
  see also optimal stopping time(s)
strike price (in American call options) 46
strong law of large numbers 18
  applications 138, 159
submartingales 19
supermartingales 19
surveillance systems, change detection in 130

terminal decision rules 66
  applications 67, 69, 82, 92, 94, 100, 178
thresholds 71
  relationship with error probabilities 74–5
transition function (of Markov process) 60
type I and type II errors 67n

uniform integrability 17
upper exit probability of random walk 78

variance parameter (Brownian motion) 28
video editing applications 1

Wald's approximations 77–8, 79, 86
Wald's identities 25–6
  applications 30, 76, 79, 85, 113, 142, 159, 161
Wald–Wolfowitz theorem 73
  applications 75, 95
  continuous-time analogy 84
    extension 90–1
Wiener measure 29
  applications 110, 144, 152

© Cambridge University Press www.cambridge.org