Quickest Detection

The problem of detecting abrupt changes in the behavior of an observed signal or time series arises in a variety of fields, including climate modeling, finance, image analysis, and security. Quickest detection refers to real-time detection of such changes as quickly as possible after they occur. Using the framework of optimal stopping theory, this book describes the fundamentals underpinning the field, providing the background necessary to design, analyze, and understand quickest detection algorithms.

For the first time the authors bring together results that were previously scattered across disparate disciplines, and provide a unified treatment of several different approaches to the quickest detection problem. This book is essential reading for anyone who wants to understand the basic statistical procedures for change detection from a fundamental viewpoint, and for those interested in theoretical questions of change detection. It is ideal for graduate students and researchers in engineering, statistics, economics, and finance.

H. Vincent Poor is the Michael Henry Strater University Professor of Electrical Engineering, and Dean of the School of Engineering and Applied Science, at Princeton University, from where he received his Ph.D. in 1977. Prior to joining the Princeton faculty in 1990, he was on the faculty of the University of Illinois at Urbana-Champaign, and has held visiting positions at a number of other institutions, including Imperial College, Harvard University, and Stanford University. He is a Fellow of the IEEE, the Institute of Mathematical Statistics, and the American Academy of Arts and Sciences, as well as a member of the US National Academy of Engineering.

Olympia Hadjiliadis is an Assistant Professor in the Department of Mathematics at Brooklyn College of the City University of New York, where she is also a member of the graduate faculty of the Department of Computer Science. She was awarded her M.Math in Statistics and Finance in 1999 from the University of Waterloo, Canada. After receiving her Ph.D. in Statistics with distinction from Columbia University in 2005, Dr. Hadjiliadis joined the Electrical Engineering Department at Princeton as a Postdoctoral Fellow, where she was subsequently appointed as a Visiting Research Collaborator until 2008.

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H. VINCENT POOR
Princeton University

OLYMPIA HADJILIADIS
City University of New York

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To our families

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Preface

Change detection is a fundamental problem arising in many fields of engineering, in finance, in the natural and social sciences, and even in the humanities. This book is concerned with the problem of change detection within a specific context. In particular, the framework considered here is one in which changes are manifested in the statistical behavior of quantitative observations, so that the problem treated is that of *statistical change detection*. Moreover, we are interested in the on-line problem of *quickest detection*, in which the objective is to detect changes in real time as quickly as possible after they occur. And, finally, our focus is on formulating such problems in such a way that optimal procedures can be sought and found using the tools of stochastic analysis.

Thus, the purpose of this book is to provide an exposition of the extant theory underlying the problem of quickest detection, with an emphasis on providing the reader with the background necessary to begin new research in the field. It is intended both for those familiar with basic statistical procedures for change detection who are interested in understanding these methods from a fundamental viewpoint (and possibly extending them to new applications), and for those who are interested in theoretical questions of change detection themselves.

The approach taken in this book is to cast the problem of quickest detection in the framework of optimal stopping theory. Within this framework, it is possible to provide a unified treatment of several different approaches to the quickest detection problem. This approach allows for exact formalism of quickest detection problems, and for a clear understanding of the optimality properties they enjoy. Moreover, it provides an obvious path to follow for the researcher interested in going beyond existing results.

This treatment should be accessible to graduate students and other researchers at a similar level in fields such as engineering, statistics, economics, finance, and other fields with comparable mathematical content, who have a working knowledge of probability and stochastic processes at the level of a first-tier graduate course. Although the book begins with an overview of necessary background material in probability and stochastic processes, this material is included primarily as a review and to establish notation, and is not intended to be a first exposure to these subjects. Otherwise, the notes are relatively self-contained and can be read with a knowledge only of basic analysis. Some previous exposure to statistical inference is useful, but not necessary, in interpreting some of the results described here.

The material presented in this book is comprised primarily of results published previously in journals. However, the literature in this area is quite scattered across both time and disciplines, and a unique feature of this treatment is the collection of these results into a single, unified work that also includes the necessary background in optimal stopping theory and probability. Although this book is intended primarily as a research monograph, it would also be useful as a primary text in a short course or specialized graduate course, or as a supplementary text in a more basic graduate course in signal detection or statistical inference.

This book grew out of a series of lectures given by the first author at the IDA Center for Communications Research (CCR) in Princeton, New Jersey, and much of the material here, including primarily the discrete-time formalism treated in Chapters 2–6, originally served as lecture notes for that series. On the other hand, most of the material on continuous-time models in Chapters 2–6, and all of the material in Chapter 7, was completed while the second author was a post-doctoral associate at Princeton University under the support of the US Army Pantheon Project.

The writing of this book has benefited from a number of people other than the authors. Among these are David Goldschmidt, former Director of CCR, who suggested and encouraged the series of lectures that led to the book, and George Soules, former Deputy Director of CCR, who provided many useful suggestions on the subject matter and presentation of this material. Early drafts of this material also benefited considerably from the comments and suggestions of the many members of the technical staff at CCR who attended the lectures there. We are very grateful to these colleagues, as well as to many other colleagues at Princeton University and elsewhere who have made useful suggestions on this treatment. Of these, we mention in particular Richard Davis of Colorado State University, Savas Dayanik and Stuart Schwartz of Princeton, Robert Grossman of the University of Illinois, Chris Heyde of the Australian National University, Kostas Kardaras of Boston University, George Moustakides of the University of Patras, and Alexander Tartakovsky of the University of Southern California. Finally, the authors are also grateful to Phil Meyler of Cambridge University Press for his encouragement of this project and for his patience in awaiting its completion.

Frequently used notation

$\mathbb{R}$ is the set of all real numbers
$\mathcal{Z}$ is the set of all integers
$\mathbb{R}^\infty$ is the set of (one-sided) sequences of real numbers
$C[0, \infty)$ is the set of continuous functions mapping $[0, \infty)$ to $\mathbb{R}$
$D[0, \infty)$ is the set of functions mapping $[0, \infty)$ to $\mathbb{R}$ that are right-continuous and have left limits