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Spaces of Kleinian Groups

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Contents

Preface	vii
Drilling short geodesics in hyperbolic 3-manifolds <i>K. Bromberg</i>	1
On topologically tame Kleinian groups with bounded geometry <i>Ken'ichi Ohshika and Hideki Miyachi</i>	29
An extension of the Masur domain <i>Cyril Lecuire</i>	49
Thurston's bending measure conjecture for once punctured torus groups <i>Caroline Series</i>	75
Complexity of 3-manifolds <i>Bruno Martelli</i>	91
Moduli of continuity of Cannon–Thurston maps <i>Hideki Miyachi</i>	121
Variations of McShane's identity for punctured surface groups <i>Hirota Akiyoshi, Hideki Miyachi and Makoto Sakuma</i>	151
Train tracks and the Gromov boundary of the complex of curves <i>Ursula Hamenstädt</i>	187
The pants complex has only one end <i>Howard Masur and Saul Schleimer</i>	209
The Weil–Petersson geometry of the five-times punctured sphere <i>Javier Aramayona</i>	219
Convexity of geodesic-length functions: a reprise <i>Scott A. Wolpert</i>	233
A proof of the Ahlfors finiteness theorem <i>Albert Marden</i>	247
On the automorphic functions for Fuchsian groups of genus two <i>Yohei Komori</i>	259
Boundaries for two-parabolic Schottky groups <i>Jane Gilman</i>	283

vi

Searching for the cusp <i>David J. Wright</i>	301
Circle packings on surfaces with projective structures: a survey <i>Sadayoshi Kojima, Shigeru Mizushima and Ser Peow Tan</i>	337
Grafting and components of quasi-fuchsian projective structures <i>Kentaro Ito</i>	355
Computer experiments on the discreteness locus in projective structures <i>Yasushi Yamashita</i>	375

Preface

This volume is the proceedings of the programme *Spaces of Kleinian Groups and Hyperbolic 3-Manifolds* held at the Isaac Newton Institute in Cambridge, 21 July–15 August 2003. It is a companion volume to *Kleinian Groups and Hyperbolic 3-Manifolds*, London Mathematical Society Lecture Notes **299**, the proceedings of a conference with the same title held at the Mathematics Institute, University of Warwick, 11–15 September 2001.

The period surrounding these two conferences has seen a series of remarkable advances in our understanding of hyperbolic structures on 3-manifolds. Many of the outstanding issues immediately preceding the Newton Institute meeting related to difficulties in extending results from manifolds with incompressible boundary to the general case. Proofs of Thurston's ending lamination conjecture and the Bers–Sullivan–Thurston density conjecture for general tame groups were announced at the meeting, and the picture was completed not long after the Newton programme, with two independent proofs of Marden's tameness conjecture. As a result, we now have a very clear understanding of the internal geometry of hyperbolic 3-manifolds, combined with an increasingly detailed, but quite intricate, picture of the topology and geometry of the associated deformation spaces of discrete groups.

The Newton Institute meeting turned out to be the international gathering at which many of these new results were disseminated. Almost all the primary contributors took part. Quite how rapid progress has been only became apparent to many of us during the meeting, which will be remembered as a milestone at which all of the new ideas were brought together.

This volume contains articles and expositions which it is hoped will give some impression of the breadth and scope involved. Contributions have been arranged bringing similar themes together, starting with topology and geometry of 3-manifolds, moving through curve complexes and classical Ahlfors–Bers theory, to computer explorations and projective structures.

The editors, who were also the organisers of the SKG programme, would like to extend thanks on behalf of all the participants to the Newton Institute for hosting us in such pleasant surroundings so conducive to mathematical interaction. We enjoyed generous funding not only from the EPSRC but also the EU, the NSF, the Leverhulme Trust, the London and Edinburgh Mathematical Societies and, through various individual grants, the JSPS. We acknowledge with thanks the support of all these bodies. Finally, we are extremely grateful to David Sanders for his skilled editorial assistance, without which this volume would not have been produced.

Yair Minsky, Makoto Sakuma & Caroline Series
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