

General index

- adapted, 51
- adjacent, 216
- adjusted, 149
- admissible, 237
- affine, 35, 36, 133
 - dimension, 133
 - hull, 35, 133
 - independence, 133, 157
- algorithmically planar, 108
- angle, 39, 55, 125
 - azimuth, 51, 52, 118, 241
 - dihedral, 43, 48, 52, 65, 176, 254
 - total solid, 175
 - zenith, 54
- annulus, 184
- apex, 66
- arc, 39, 40
 - geodesic, 40
- arccosine, 32
- arclength, 39, 40
- arctangent, 31
 - \arctan_2 , 31
 - derivative, 31
 - near 0, 32
- Aristotle, 150
- azimuth, 51, 52, 57, 58, 118, 241
- azimuth cycle, 51, 57, 58, 114
- ball, 67
- ball, open and closed, 63
- Bezdek, K., 252
- bisector, 147
- blade, 37, 113
- boundary
 - relative, 133
- canonical function, 97
- cardinality, 237, 241
 - finite, 70
- carrier, 105
- Cauchy–Schwarz inequality, 34, 39, 41
- Cayley–Menger
 - determinant, 37, 68
- cell, 167, 169
- cell cluster, 181, 182
- circular cone, 62
- circular fan, 197, 198
- circumcenter, 157
- circumradius, 157
- claim (*italic format of small claims*), xii
- closed, 135
- closed ball, 63
- closure, 133
- clusters, 181
- codimension, 152
- collinear, 37
- combinatorial component, 77, 117
- complement, 88
- component
 - combinatorial, 77, 87, 127
 - topological, 126, 131, 138
- computer calculation, 251
 - 5202826650 a, 234
 - BIEFJHU, 191
 - KCBLRQC, 246, 247
 - OXLZLEZ, 183
 - TNVWUGK, 256
 - TSKAJXY, 182
 - TVAWGDR, 240
 - UKBRPFE, 192
 - UPONLFY, 227
 - WAZLDCD, 192
 - notation, xii

- cone
 - right-circular, 65
- conforming, 126, 128, 243
- conic cap, 66, 67
- connected, 77, 117, 120, 127, 138
 - combinatorial component, 117
 - topological component, 117, 118, 120, 123, 124
- constraint system, 216
- contact fan, 238
- contour
 - loop, 86
 - Moebius, 88
 - path, 86
- contravening, 17, 20, 243
- convex, 36, 126, 133, 188
- convex hull, 36, 162, 255
- coordinate systems, 52, 54, 55
 - cylindrical coordinates, 52
 - polar coordinates, 51
 - spherical coordinates, 54
- coplanar, 37, 65
- corrected volume, 149
- correction term, xiii
- cosine, 27
 - derivative, 28
 - law of cosines, 40
 - roots, 29
 - series definition, 27
 - spherical law of cosines, 45, 47
- cover, 222
- critical edge, 181
- cross product, 42
- cyclic
 - list, 86
 - permutation, 55, 57, 74, 196
 - set, 57, 58
- cylindrical coordinates, 52
- D-cell, 254
- dart, 74, 116, 118, 119, 123, 138
 - degenerate, 77
 - isolated, 116
 - nondegenerate, 77
 - set, 76
- decomposition
 - Delaunay, *see* decomposition
 - Marchal, 18, 167
 - Rogers, 150, 253
 - Voronoi, 9, 146, 167, 252
- deformation, 203, 224
- degenerate, 77, 79
- Delaunay, *see* decomposition
- determinant, 38
 - Cayley–Menger, 37, 68
- Dhammadapa, 77
- diagonal, 213
- diagonal cover, 222, 224
- dihedral, 94
- dimension, 133
- disjoint sum decomposition, 119
- dodecahedral conjecture, 252
- dot product, 34
- double join, 93, 238
- ear, 217
- edge, 77, 133, 134
 - fan, 114
 - graph, 114
 - length, 38, 43, 254
 - map, 74
 - walkup, 79
- equivalence relation, 76
- Euclidean space, 33
- Euler characteristic relation, 84
- Euler’s formula for solid angle, 68
- eventually, *see* radial
- exposed, 254
- extension, 98
- extremal edges, 175
- extreme point, 133, 134, 175
- face, 77, 133–135, 238
 - attribute, 128
 - map, 74
 - walkup, 79
- face-centered cubic, *see* FCC
- facet, 133, 135
- fan, 112, 113, 118, 124, 137
 - contact, 238, 241
 - local, 195, 198, 203, 206, 207, 211
 - standard, 238
- fan, 113
- FCC, 7, 10, 148
 - compatible, 149, 179
 - pattern, 10
- Fejes Tóth, L., 253
- final, 108
- fixed point, 77
- flag, 96, 97
- Flyspeck, 257
- frame, 51, 57
- free, 225
- frustum, 65, 67
- fully surrounded, 121, 243

266

generation, 95
 generic fan, 197, 198
 Girard's formula, 48, 68, 125, 188
 Google Code project hosting, xii
 graph record, 107
 great circle, 46
 group, 164
 half-plane, 37, 191
 half-space, 37, 65, 125, 147, 191
 open, 37
 Harriot, T., 48
 HCP, 10, 148
 pattern, 10
 Heron's formula, 41
 hexagonal layer, 11
 hexagonal-close packing, *see* HCP
 HOL Light, xiii
 hull
 affine, 35, 133
 hypermap, 72, 74, 112, 114, 118, 119, 123,
 124, 127, 138
 algorithm, 96
 connected, 127
 contravening, 241
 dihedral, 94
 opposite, 250
 plain, 83
 planar, 89, 192
 planar index, 84
 restricted, 96
 simple, 127
 subquotient, 92
 tame, 235, 236
 hyperplane, 133
 I (use of personal pronoun), xii
 interior, 196
 point, 138
 relative, 133
 interior point, 133
 Isabelle/HOL, 95
 isolated, 116, 241
 isomorphism
 hypermap, 91
 proper, 105
 tame hypermap classification, 251
 torsor, 216
 Java, 95
 Jordan curve theorem, 112
 Kepler conjecture, 148, 150, 251
 latitude, 54

General index

lattices, x
 law of cosines, 14, 40
 law of sines, 42
 lead into, 120
 length, 152
 level, 223
 line, 37
 linear stretch, 63
 linearly independent, 158
 list, 75
 listing, 105
 local fan, 195
 local inequality, 255
 localization, 194, 196
 longitude, 26, 52
 loop, 86, 238
 lunar fan, 197, 198
 lune, 64, 65
 map, 105
 Marchal cell, 18, 167
 Marchal, C., 167
 marked hypermap, 98
 Mathematica, 95
 measurable, 62
 measure, 61, 62, 146
 Lebesgue, 62
 merge, 79, 81
 metric space, 117
 minimal counterexample, 223
 ML, 95
 named property
 angle, 195
 bijection, 126
 bound a, 237
 bound b, 237
 bound d, 237
 cardinality, 113
 circular fan, 197
 diagonal, 126
 dihedral, 195
 face, 195
 face count, 238
 face size, 238
 fan, 195
 generic fan, 197
 half-space, 126
 intersection, 113
 linear stretch, 63
 lunar fan, 197
 no double joins, 238
 no loops, 238

- node count, 238
- node size, 238
- node types, 238
- nondegenerate, 238
- nonnegative, 62
- nonparallel, 113
- null difference, 62
- null set, 62
- origin, 113
- planar, 238
- primitives, 63
- simple, 238
- solid angle, 126
- surroundedness, 126
- translation, 63
- union, 62
- wedge, 195
- weights, 238
- natural numbers, 26, 27
- negligible, 148, 149, 179
- node, 77, 134, 237
 - map, 74
 - properties, 245
 - type, 236
 - walkup, 79
- nondegenerate, 77, 238
- nonreflexive, 195
 - local fan, 198
- norm, 34
- normal family, 91, 92
- null set, 62, 254
- open ball, 63
- open half-space, 37
- opposite, 250
- orbit, 77, 80
- order, 74
 - lexicographic, 55
 - total, 55
- orthogonal frame, 54
- orthogonality, 43
- overloaded, 151
- packing, 112, 145, 146
 - finite, 241
- parallel, 37
- parallelepiped, 37
- partial perimeter, 211
- partition, *see* decomposition
- path, 75, 77, 86
 - injective, 76
 - maximal, 92
- path connected, 118
- pencil and pen heuristic, 96
- perimeter, 211
 - nonreflexive local fan, 211
- periodicity, 29
- permutation, 73, 164
- plain, 77
- planar, 21, 83, 112, 114, 127, 238
 - algorithmic, 108
 - graph, 21, 72
 - index, 84
 - map, 72
- plane, 37, 62, 77, 121
 - graph, 21, 72, 74
- polar
 - coordinate, 120
 - coordinates, 51
 - cycle, 55, 57
 - fan, 209
 - triangle, 46
- polygon, 185, 188
- polyhedron, 134, 135, 137, 138, 189, 192
- polysemes (face, edge, node), 134
- positive, 51
- primitive region, 62, 66
- proper, 133
- protracted, 213
- quadrilateral, 236
- quotient dart, 92
- radial, 64
 - eventually, 63, 70
 - set, 61, 63
- real analysis, 27
- real arithmetic, 27
- rearrangement, 164, 173
- record, 108
- rectangle, 67
- reflex angle, 195
- regular dodecahedron, 255
 - surface area, 255
 - volume, 253, 255
- relative boundary, 133, 135
- relative interior, 133
- restricted hypermap, 96
- right-circular cones, 65
- Rogers, *see* decomposition
- rotation, 55
- saturated, 15, 146
- set (of a list), 105
- simple, 77, 238
- sine, 27

268

law of sines, 42
 series definition, 27
 slice
 fan, 206, 207
 torsor, 221
 solid angle, 63, 64
 solid triangle, 65, 67
 sphere, 62, 117
 sphere packing problem, 150
 spherical
 coordinates, 54, 120
 law of cosines, 45, 47
 triangle, 45
 triangle inequality, 59
 split, 79
 stable, 216
 standard, 213
 standard fan, 20, 238
 standard main estimate, 215
 steps, 75
 straight fan node, 198, 226
 strong dodecahedral conjecture, 252
 sublist, 75, 86
 subquotient, 91, 92
 surface area, 253
 exposed, 254
 surrounded, 241
 symmetric difference, 62
 tame, 17, 21, 235, 238
 contravention, 243
 hypermap, 21, 236
 tangent, 30
 target, 236
 Tarski arithmetic, 27, 66
 tetrahedron, 43, 65, 67
 topological component, 117
 torsor, 216
 total solid angle, 175
 transfer, 224
 transform, 101
 translation, 63
 triality relation, 75
 triangle, 236
 Euler, 49
 spherical, 45, 48, 59
 triangle attributes, 125
 triangle inequality, 35, 41
 trigonometry, 25
 addition formula, 28
 arccos, 33
 arctan, 33

General index

circle identity, 28
 identities, 28
 inverse, 32
 law of cosines, 40
 law of sines, 42
 periodicity, 30
 spherical, 45
 tangent, 30
 type, 236
 unit list, 76
 vector, 33
 addition, 34
 cross product, 42
 dot product, 34
 norm, 34
 projection, 43, 57
 scalar multiplication, 34
 subtraction, 34
 zero, 33
 vector space, 34
 vertices, 134
 visits, 76
 volume, 63
 primitive, 63
 Voronoi, *see* decomposition
 walkup, 78, 79, 83
 degenerate, 79
 double, 81
 we (use of personal pronoun), xii
 weakly saturated, 189
 wedge, 64, 65, 67, 69, 118
 weight, 182, 238
 admissible assignment, 237
 assignment, 236, 237
 total, 237
 zenith, 54

Notation index

- \sim (equal up to positive scalar), 53, 209
 \sim (equivalence relation), 76
 $\binom{n}{k}$ (binomial coefficient), 175
 $[Y]$ (topological components), 118
 $[\mathbf{u}, \mathbf{v}, \mathbf{w}] = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$, 211
 $*[\mathbf{v}, \mathbf{w}]$ (slicing a fan), 206
 \sphericalangle (dart angle), 196, 206
 $\#c$ (number of components), 77
 $\#h$ (number of orbits), 77
 \cdot (dot product), 34
 $*$ (wildcard symbol), xii
 $*'$ (polar local fan (V', E', F')), 209
 $*^c$ (complement), 88
 \times (cross product), 42
 \perp , 159
 \rightarrow_p (input-output hypermap-generating relation), 108
 A (Voronoi cell face area), 253
 A (index set), 246
 A (plane), 37
 A (set of faces), 246
 A (set of triangles), 237, 247
 ABC (triangle), 46
 $A(\mathbf{u}, \mathbf{v})$ (bisector), 147
 $A_+(\mathbf{u}, \mathbf{v})$ (half-space), 147
 aff (affine hull), 133
 $\text{aff}_\pm, \text{aff}_\pm^0$, 36, 65
 $a = 0.63$ (tame parameter), 247
 a_D (dodecahedral parameter), 255
 arc , 40
 \arccos , 32, 33
 \arctan , 31, 33
 \arctan_2 , 31
 arc_V , 39
 azim , 52, 58, 118
 $\text{azim}(H, \mathbf{v})$, 206
 $B(\mathbf{v}, r)$ (open ball), 17, 63, 146
 $\bar{B}(\mathbf{v}, r)$ (closed ball), 63
 b (tame parameter), 237
 b_D (dodecahedral parameter), 255
 \mathcal{B} , 184
 \mathcal{B}_s , 217
 C (set), 63
 C (wedge), 69
 $C(S)$ (cell-like subset of \mathbb{R}^3), 170
 C_\pm, C_\pm^0 (blade), 113
 c_i (ranking functions on a packing), 242
 CAP , 66, 70
 $c_{\text{stab}} = 3.01$, 216
 CL (cell cluster), 182
 cl (closure), 133
 cluster , 182
 conv , 36
 \cos , 27
 D (dart set), 74, 92, 116
 D (determinant), 38
 D (spherical disk), 190
 D_k (D -cell), 254
 d (real parameter), 237, 250
 $d(s, \mathbf{v})$, 220
 $d(\mathbf{u}, \mathbf{v})$ (metric on \mathbb{R}^3), 59
 d_j (truncation of lists), 152
 dih , 43, 52, 65, 70, 176, 254
 dih_i , 213
 dih_V , 43
 dim aff (affine dimension), 133, 151
 E (edge set of a fan), 113
 E_P (edge), 137
 E_{etc} (edge set of contact fan), 238
 E_{std} (edge set of standard fan), 238

- E' (polar edge set), 209
 $E(\mathbf{v})$ (set of edges adjacent to \mathbf{v}), 114, 241
 $E(X)$, 175
 EC (critical edges), 182
 E (frame), 51
 equi (intersection of spheres), 170
 e (edge map), 74
 \mathbf{e}_i (orthonormal vectors), 52
 F (hypermap face), 94, 125, 249
 F (polygon), 246
 F' (polar face), 209
 \mathbf{F} (subquotient bijection), 92
 f (function name), 57
 f (face map), 74
 FR (frustum), 65, 70
 G (isomorphism of hypermaps), 91
 G (negligible function), 149
 g (function name), 183
 g (graph record), 108
 g (triangle area), 188
 (H, τ) , 245
 H (hypermap), 84, 96, 243, 245, 246, 251
 H/\mathcal{L} (subquotient), 92
 h (circumradius), 160
 h (cylindrical coordinate), 52
 h (half-edge length), 176
 h (permutation), 77
 $h_+ = 1.3254$, 177
 $h_- \approx 1.23175$, 181
 $h_0 = 1.26$, 181, 213
 hyp (hypermap), 116, 238
 I (identity map), 73
 I (real interval), 203
 I (torsor), 216
 $I[p, q]$ slice (of a torsor), 221
 J , 216
 k (cardinality of a face), 128, 249, 250
 L (dart path), 76
 L (linear function), 181, 213, 244
 \mathcal{L} (normal family of loops), 92
 ℓ (level function), 223
 $\mathcal{L}(V)$ (estimation of a packing), 17, 235, 243, 251
 $\ell_H(x)$ (listing of dart x), 105
 M (Marchal's quartic), 177
 m (face map exponent), 99
 $m_1 \approx 1.012$, 177
 $m_2 \approx 0.0254$, 177
 $\text{map}(\phi, \ell)$ (map of a function over a list), 105
 N (integer invariant of a fan), 128
 $\mathbb{N} = \{0, 1, \dots\}$, 27
 n (integer variable), 58
 $\text{node} : D \rightarrow V$ (node of a dart), 119, 196
 n (node map), 74
 O (Landau's big O), 180
 $O(h, x)$ (orbit of x under h), 80, 105
 P (dart path), 76, 90
 P (polyhedron), 135
 p (Euler solid angle numerator), 49
 p (face map exponent), 99
 p (trigonometric expression), 47
 $\mathbf{p} \in \mathbb{R}^3$, 62
 per (perimeter), 211
 (p, q, r) (node parameters), 236, 246
 p_v , 236
 R (Rogers simplex), 154
 \mathbb{R} (field of real numbers), 27
 r (polar, cylindrical, and spherical radius), 32, 51, 52, 54, 65
 rcone , rcone^0 , 65
 \mathbb{R}^N , 33
 reg (area of regular spherical polygon), 191
 ri (relative interior), 133
 S (finite subset of \mathbb{R}^3), 157
 S (flag, set of darts), 97
 $S(H, L, x)$ (flag set), 97
 $S^2(r)$ (sphere of radius r), 117, 204
 s (constraint system), 216
 s (real variable), 124
 seed_p , 108
 set (of a list), 105
 \sin , 27
 S_{main} (main estimate constraint systems), 217
 $\text{sol}_0 = 3 \arccos(1/3) - \pi$, 177, 213, 244
 sol (solid angle), 64, 254
 soly (solid angle as a function of edges), 254
 $\text{sol}(V, E, F)$ (formal solid angle), 208
 split_c , split_f , 85
 surf (surface area), 254
 Sym (symmetric group), 164, 166
 T (regular tetrahedron), 150
 T (rotation), 56
 T (transform), 101
 T_v (linear stretch), 63
 t (real variable), 246
 \tan , 30
 TET (tetrahedron), 65
 $\text{tgt} = 1.541$, 236, 244, 247
 TRI (solid triangle), 65, 66

- tsol , 175
 U_x (topological component), 120
 U_F (topological component), 123, 125, 243
 (V, E) (fan), 113, 238
 V (packing), 145, 149, 238
 V (subscript marking vector functions), 40, 43
 V' (packing), 241
 V' (polar node set), 209
 $V(X) = V \cap X$, 175
 $V \subset \mathbb{R}^n$, 36
 $V(\mathbf{p}, r) = V \cap B(\mathbf{p}, r)$, 146
 \mathcal{V} (configuration space), 242
 \mathcal{V}_H (configuration space), 251
 $\mathbf{v} \in \mathbb{R}^3$, 37, 39, 65, 69, 237
 $\mathbf{v}(t)$ (deformation of \mathbf{v}), 203, 225
 vol , 62
 vol_D (volume of dodecahedron), 253
 W, W^0 (wedge), 64, 65
 W_F (topological component of a facet), 138
 W (walkup), 79, 85
 W_{dart}^0 (wedge), 118, 120, 206
 $W_{\text{dart}}^0(F, \mathbf{v}), W_{\text{dart}}(F, \mathbf{v})$, 196
 W_{dart} (closure of W_{dart}^0), 118
 $\mathbf{w} \in \mathbb{R}^3$, 39, 65
 wt (weight), 182
 X (measurable set), 62
 $X(V, E) \subset \mathbb{R}^3$ (union of fan blades), 118
 x (dart), 78, 79, 246
 (x, y) (Cartesian point), 32
 Y (measurable set), 62
 $Y(V, E) \subset \mathbb{R}^3$ (fan complement), 118
 y (dart), 99
 y_D (dodecahedral parameter), 255
 z (dart), 99
 z (real variable), 246
 α (angle), 48, 70, 188
 β (angle), 48, 188
 β, β_0 (bump), 182
 Γ , 182
 γ (angle), 40, 53, 188
 γ (packing inequality), 178
 Δ , 37, 38, 47, 51
 $\delta(V, \mathbf{p}, r)$, 149
 $\delta \in \mathbb{R}$, 123
 $\delta \in \{0, 1\}$, 223
 $\epsilon \in \mathbb{R}$, 118, 123
 ϵ (edge of a fan), 113
 η (circumradius), 241
 θ (polar, cylindrical, and spherical angle), 32, 51–54, 56, 69
 ι (planar index), 84
 v_D (dodecahedral function), 255
 ξ (cell parameter), 168
 ρ (permutation on nodes of a local fan), 196
 ρ (permutation), 164
 ρ (rotation of lists), 106
 ρ_0 (real-valued function), 213
 σ (azimuth and polar cycle), 55–58
 $\sigma = \pm 1$, 220
 τ (weight assignment), 213, 237, 244–246, 250
 τ^* , 220
 τ_{tri} , 213, 227, 228, 233
 $\tau_0 \approx \text{tgt}$, 177
 v (polynomial), 38, 41, 47
 $\check{\varphi}_{\text{can}}$ (canonical function), 97
 φ (path), 124
 ϕ (proper isomorphism between sets of lists), 105
 ϕ (zenith), 54
 ψ , 52, 56
 Ω (Voronoi cell), 146, 253
 $\Omega(V, W)$ (intersection of Voronoi cells), 151
 ω (extreme points of Rogers simplex), 154