1   The limits of personalism

THE OBJECT OF THIS BOOK is to develop a philosophical theory about statistical probability. I call statistical probability 'chance' for brevity and to mark it off from inductive probability. Statistical probability is what is meant in saying of a coin either

    H_1:   The probability of coin a landing heads when tossed is p_1
or
    H_2:   The probability of coin a landing heads when tossed is p_2.

Inductive probability is what is meant in saying that on the evidence available of symmetry, results of tosses etc., that

    H_1 is more probable than H_2.

My main concern is thus with what Carnap (1962) calls 'probability$_2$' as opposed to what he calls 'probability$_1$'. But I do not share his view (1962, §9; 1955) that these are distinct senses of an ambiguous term 'probability' (cf. Ayers, 1968, pp. 42–50). Nor do I accept his frequency account of 'probability$_2$'.

    The theory of this book is that some chance statements such as H_1 and H_2 can be made true by things having a certain dispositional property. Following Popper (1957, 1959a), I call this property 'propensity'. To pursue the example, the *bias* of a coin (or its un-biasedness) is a propensity which makes true some such statement as H_1 of the chance of it landing heads when tossed. Naturally there are many other and more serious instances of propensities. Before discussing them in detail it is desirable to indicate more clearly what this propensity theory of chance asserts and how far it absorbs, denies and supplements other theories of probability.

    Propensity theories have been presented before. Such a conception of statistical probability was suggested before Popper by Peirce (1931, volume 2, §664) and has later been developed by Hacking (1965) and adopted by Levi (1967). It will be obvious in the sequel how much the present account owes to their work. But it differs from these earlier accounts in a number of ways. First I attend more to what kind of item a propensity could be and how like it is to other scientific properties.

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Hacking is in contrast more concerned with statistical inference itself than with the feature of the world that, as I hold, sometimes makes such inference reasonable. Popper and Levi are likewise sketchy on what propensities are and they do not draw out the detailed consequences of their views. My intention is, by so drawing out consequences, to present at least a clear and detailed view for criticism if not to convert adherents of other views.

Secondly however, I hope also to show that much obvious truth in other views is herein accommodated. The propensity theory is not a comprehensive and exclusive new theory of probability in general but an account of one kind of objective probability statement. It certainly conflicts with such limiting frequency views as those of von Mises (1957) and Reichenbach (1949), although it can be largely reconciled to Braithwaite's (1953) more sophisticated frequentism. On the other hand, propensity theory can observe at least a non-aggression pact with accounts of logical probability statements as in Carnap (1962) and Jeffreys (1961). And it is positively a feature of this version of propensity theory to base itself explicitly on the personalist theories of Ramsey (1926), de Finetti (1937), Savage (1954) and Jeffrey (1965). Personalists have admittedly tended to accompany their theories with a view of probability statements as merely subjective. But that is an incidental defect of personalism which it is my principal aim in this chapter and the next to expose and remove. To this end I generally reserve 'sub-jective' and 'subjectivism' hereafter for the doctrine I reject, that there can be no objective probabilities. I use 'personalism' to refer to the accounts of chance statements as expressing "partial beliefs", which broadly I accept.

The relation between the propensity and personalist theories is this. According to the latter the making of a probability statement expresses the speaker's "partial belief" in whatever he thereby ascribes proba-bility to, say that a coin a will land heads when tossed. Knowledge of the coin's propensity on the present theory is what in suitable circumstances makes reasonable the having of some particular partial belief in the outcome of the toss. The chance of the coin falling heads when tossed is then the measure of that reasonable partial belief.

It is not usual to base an account of objective probability on a concept of partial belief. It may well be asked why one should do so. Kneale, discussing relational probability rather than chance, puts the question rhetorically (1949, p. 13):

A man who knows that the evidence at his disposal justifies a certain degree of confidence in proposition *A* must know that the evidence probabilifies *A* to a certain degree; for it is only so that the evidence can justify any degree of confidence. But if a man who has a rational opinion knows all this (even although he may not have the terminology in which to state it explicitly), why need we say in addition that he has a degree of confidence in *A* which somehow corresponds with the degree to which the evidence probabilifies *A*? Can we not content ourselves with the assertion that rational opinion is the knowledge that the available evidence probabilifies a proposition to a certain degree?

One might similarly ask of chance: can we not analyse full belief that the chance of heads on a coin toss is $\frac{1}{2}$ without reference to some supposedly corresponding partial belief that the coin will land heads? The reason for denying this is the fact to which Kneale himself draws attention (p. 18) "that knowledge of probability relations is important chiefly for its bearing on action". It follows as Kneale says (p. 20) that "no analysis of the probability relation can be accepted as adequate... unless it enables us to understand why it is rational to take as a basis for action a proposition which stands in that relation to the evidence at our disposal". Similarly with chance. It must follow from our account that the greater the known chance of an event the more reasonable it is to act as if it will occur. What can intelligibly come by degrees, however, turns out not to be reasonableness so much as a tendency to act as if an event would occur. This concept of a quantitative tendency to action is just that of partial belief as it has been developed by personalists. It is thus available to provide in our account of chance that necessary connection with action on which Kneale rightly insists. A great difficulty facing other objective accounts of chance, notably the frequency theories, has been to build such a connection subsequently on to their entirely impersonal foundations (see e.g. Braithwaite, 1966). In proceeding differently we shall of course later need to show that no properly hallowed Humean doctrine is denied.

The other advantage which propensity theory has in basing itself on partial belief is, curiously enough, over subjectivism in being less open to charges of idealisation. We shall see that personalists credit people with partial beliefs whose measure is a probability. Many of their arguments, however, are presented as both normative and limited: e.g. that if a man has certain partial beliefs he can be made to lose money should they not be probabilities. Personalists take their theory to be refuted neither by a person lacking partial beliefs altogether nor by their failing to satisfy such personalist constraints of "coherence".

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So de Finetti (1937, p. 111): '[Personalist] probability theory is not an attempt to describe actual behavior; its subject is coherent behavior, and the fact that people are only more or less coherent is inessential.' But de Finetti does not disdain empirical support even on the same page as he disdains empirical refutation: 'The notion of probability which we have described is without doubt the closest to that of "the man in the street"; better yet, it is that which he *applies every day in practical judgments . . .* What more adequate meaning could be discovered for the notion?' (my italics).

   But however much or little it matters to personalists that people have suitable partial beliefs, it matters less to propensity theory. Our subject is not partial belief itself but that feature of the impersonal world, namely propensity, knowledge of which can make some partial beliefs more reasonable than others. The actual existence of people with such partial beliefs is as immaterial to propensity theory as that of radio-elements and dice is to games and decision theories. Given that the latter theories provide a suitable concept of partial belief, propensity theory is no further concerned with it in detail than thermodynamics is with subjective feelings of warmth. Subjectivism, on the other hand, offers partial belief as a surrogate for the objective chance that statistical sciences ostensibly refer to. It is very material to one who would dismiss objective chance as a fiction that mathematically suitable partial beliefs exist to take its place.

   Nevertheless, even for our purposes more must be said of partial belief if propensity is to be intelligibly characterised in terms of it. It may be as subjective a notion but it is not as plain to the senses as are feelings of warmth. More argument is needed to show that the concept of partial belief will serve our turn; in particular that partial belief is suitably related to full belief, e.g. in a coin landing heads, and to behaviour consequent thereon.

BELIEF AND PARTIAL BELIEF

   Often enough, my cat's behaviour makes it clear to me that he believes he is
   about to be fed.

                                                             Jeffrey, 1965, p. 59

It is a commonplace of personalism that in ascribing a probability one expresses a certain attitude which may reasonably be referred to by such terms as 'doubt', 'expectation', 'degree of conviction' or 'partial

belief'. I mostly use 'partial belief' as being the most common and because of the connection to be here discussed with full belief. Similarly, I use 'belief' rather than 'believing', by analogy with 'partial belief'. There would be advantages in keeping 'belief' for what is believed (a proposition) and using 'believing' for the attitude a man has who believes (cf. Russell, 1921, pp. 232–3; Braithwaite, 1932–3, 'The Nature of Believing'; Wittgenstein, 1958, pp. 190–2). But such usage is un-common enough for it to mislead and I do not wish to add 'partial believing' to an already overloaded vocabulary.

It is usual to take propositions rather than events on the one hand or sentences on the other as the sole objects of belief and partial belief. Cases of belief or partial belief in events or states of affairs can be trivially redescribed. If I believe or partly believe in a coin landing heads I believe or partly believe in the truth of the proposition that it does so. Similarly with belief or partial belief in a state of affairs such as a coin being biased. On the other hand, we do not care what sentence (English, French or whatever) is used to state that the event occurs or state of affairs obtains. It does not, moreover, seem necessary either to belief or to partial belief that the believer have any such sentence of his native language in mind. A believer does not even seem to need a language at all if Jeffrey's cat is a fair example (p. 4 above). It likewise does not seem senseless to ascribe partial belief in his master's front door entry to a dog running excitedly from front door to back as his master approaches the side of the house. Hence for the present I follow Jeffrey (1965, pp. 48–59) in taking propositions rather than sentences as the objects both of belief and of partial belief. In so doing I do not mean or need to insist on the irreducible existence of propositions as bearers of truth. On that large topic I wish to imply no fixed view. But the traditional termin-ology here conveniently unifies diverse objects of belief and partial belief and begs fewest questions of linguistic competence.

Assuming belief and partial belief to be diverse attitudes towards propositions, what is the relation between them? There clearly is a close relation and we may at least constrain the less by the more familiar notion even if we cannot thereby completely define it.

The ordinary concept of belief is qualitative. One believes a proposi-tion q or not; the matter does not admit of degrees. Similarly with disbelieving q, which I take to be believing $\sim q$. In between it seems clearly possible to have a definite attitude towards a proposition which is yet neither belief nor disbelief but is partial belief. If partial belief is a

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quantity, it is obviously bounded by belief and disbelief. One extreme of partial belief will be a state of belief, the other a state of disbelief. Certainly less than extreme values will also count as belief. Not all one's beliefs are equally certain and a little doubt may yet fall short of agnosticism. We cannot then take absence of belief as a necessary condition of partial belief. Nor is there any point in asking just where partial belief shades into full belief. It is enough that it must do so somewhere and that our account of both concepts must allow for this necessity.

If absence of belief is not necessary for partial belief, it is even more clearly not sufficient. Lack of belief in q may signify merely a lack of any attitude whatever towards q. Even if one requires also that q be "entertained" (*cf.* Braithwaite, 1932–3, p. 132), it is not clear that a man *must* believe, disbelieve or partially believe every proposition he entertains. To say

'I don't know what to believe'

or

'I don't know whether I believe it or not'

of some topic or proposition is not really to report ignorance of one or other, presumably subconscious, attitude. It is rather to report that the speaker, although conscious of the question, is aware of being unable to come to any of these definite attitudes towards it.

In any case, we shall see later (p. 14) that the notion of entertaining a proposition is both too unclear in itself and too much tied to consciousness to be an acceptable prerequisite of belief. It is no clearer in the case of partial belief for which, for similar reasons, consciousness can hardly be essential if it is not so for belief. Certainly Jeffrey's cat and our own uncertain dog cannot as plausibly be credited with consciously entertaining propositions as they can respectively with believing and with partly believing them. Entertaining q is thus neither an effective nor an intuitively acceptable supplement to the definition one might have hoped for, namely of partial belief in terms of the absence of full belief.

Nor can appeal to the theoretical nature of partial belief make such a definition acceptable. No doubt games or decision theories may often give adequate theoretical grounds for ascribing to a person partial beliefs of which he is quite unaware. But this cannot be assumed to work by definition in every case, even taking partial belief to be a disposition and making every appeal to our unconsciousness of most of

our own dispositions. It will be granted by the theory that some propositions fail to induce any relevant disposition in a person simply because they have not come, even subliminally, to his notice. (This is not of course to deny that a man may be disposed to believe such a proposition; but that is another matter.) Even some propositions that a man has explicitly thought about, however, may still, as we have just seen, fail to produce in him any awareness of belief or of partial belief. One must not then beg the question by automatically postulating the corresponding unconscious dispositional state. It may still be that the relevant situation would call forth none of the appropriate displays of behaviour on his part. And that in turn must not be ruled out by counting every possible piece of behaviour extracted by compulsion in such a situation as a sign of some such disposition. Measuring partial belief by forcing a man to choose betting odds, for example, presupposes that he has a partial belief which the chosen odds measure. His forced choice does not itself show that he has any such disposition, because he is forced to choose whether he has or not. If he lacks any disposition, he will have to choose odds at random. That a man can always be forced to choose odds in no way shows that he always has even subconscious partial beliefs. An IQ test will analogously show what IQ a child has if it has any. But the mere fact that the test can always be applied is no answer to a sceptic who denies the existence of any such mental capacity as IQ. Like IQs, partial beliefs cannot just be stipulated into existence.

I conclude that partial belief, however highly theoretical a disposition it may be, cannot be defined in terms of full belief, with or without appeal to consciousness. Partial beliefs form a distinct family of attitudes whose existence, nature and relation to the full beliefs that are their bounds must be separately argued for.

THE NATURE OF PARTIAL BELIEF

That there is some such thing as partial belief may reasonably be inferred from common usage. Venn puts the case well (1888, p. 139):

> There is a whole vocabulary of common expressions such as, 'I feel almost sure', 'I do not feel quite certain', 'I am less confident of this than of that', and so on. When we make use of any one of these phrases we seldom doubt that we have a distinct meaning to convey by means of it. Nor do we feel much at a loss, under any given circumstances, as to which of these expressions we should employ in preference

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to the others. If we were asked to arrange in order, according to the intensity of the belief with which we respectively hold them, things broadly marked off from one another, we could do it from our consciousness of belief alone...

So far so good, but of what does "our consciousness of [partial] belief" consist? How in particular can so seemingly subjective a feeling of doubt or degrees of confidence or certainty be compared in intensity between one person and another?

At first sight partial beliefs are only subjective in the irrelevant sense that X and Y, with partial beliefs q_X and q_Y of different strengths in some proposition q, do not contradict each other merely by saying so. It is still an objective matter *that* X and Y have these partial beliefs. Y need not share X's partial belief q_X but if he denies that X has it he is objectively wrong.

If partial belief were a feeling, a merely introspectible state perhaps like a sensation of some kind, this appearance of objectivity could be misleading. The comparison is worth pressing in fact, in order to make clear that partial belief is not such a feeling but is rather a publicly detectable disposition. This is indeed widely assumed but it has not been widely argued.

Suppose first that X alone could know, by introspection, what his partial beliefs were. Now try to suppose that on this basis he could accurately report the strength of his partial beliefs. These assumptions still seem to provide no public method of correlating X's reports with the similarly couched reports of other people. To suppose that Y could be wrong, or even right, about X's partial beliefs now lacks clear sense. He could predict that X would use the expression 'q_X' to report his partial belief and could certainly be right or wrong about that. But he would seem unable to know either what X applied 'q_X' to or whether he, Y, would apply the same term to a similarly strong partial belief. As for the terms Y uses to report the strength of his own partial beliefs, he would have no way of telling if they were rightly applied to X. He would effectively be unable either to assert or to deny the accuracy of X's reports. The rest of us, not being X either, would be similarly impotent. In short, second and third person ascription of partial belief would be pointless and its apparent objectivity spurious.

Arguments like this are now widely recognised to be ineffective in showing sensations to be incommunicable; but the standard reply to them is not available in the case of partial belief. The point is that even if a sensation is private, the terms in which it is reported are not. They

can for instance also be applied on the basis of standard causes of the sensation being present, which is an entirely objective matter. My confidence that my present visual sensation of sunlit grass is rightly reported as being like yesterday's similarly caused sensation relies heavily on my knowledge that it *is* similarly caused. Through such appeal to standard causes we can even give objective sense to such remarks as

'X sees yellow more green than Y does'

by getting X and Y to match a standard yellow light against various standard mixtures of red and green light (Gregory, 1966, p. 127). The tests of matching are of course objective behavioural tests of ability to discriminate such as are used to test and measure colour blindness. We can then afford to concede the obscure claim that X and Y cannot compare absolutely the sensations they respectively receive from the same standard sources of coloured light. It is enough that they can match other colours objectively against such public standards and so describe and compare their visual sensations in these public terms.

Could we not similarly assess X's introspectible feeling of partial belief by getting him to match it with that induced by some standard source of it? It may, that is, be as senseless (or at least as immaterial) as it is with visual sensations to remark that we cannot compare absolutely the states of mind of X and Y induced by some standard cause of doubt. So long as doubt can be matched against what some standard causes induce we may have all that is needed for objective comparison of felt partial belief, as we do with seen colour.

Up to a point, indeed, the analogy holds, and has been used to characterise degrees of partial belief. Suppose X to be in doubt about the truth of a proposition q which will be settled by some agreed future observation. Let it be one of the usual propositions about the result of throwing a die or that a man dies or that some atom decays in the next year or day. Then X is offered the following choice of bets: a fixed prize either if q turns out true or, alternatively, if a standard coin that everyone agrees to be unbiased lands heads on a given toss. If X's partial belief in q makes him indifferent for all prizes which of these prospects of gain he will take, one may surely conclude that whatever it feels like to him it matches in the relevant respect his partial belief in the standard coin landing heads. The coin is moreover a very plausible inducer of a standard doubt. For X to believe it unbiased is by the same test for him to have equal partial belief in it landing heads and tails; he would

presumably feel indifferent between getting the prize on the one and getting it on the other.

This test is in fact appealed to by Ramsey (1926, p. 177) to show *inter alia* that partial belief is objectively measurable. Where it differs from the colour matching test is that the latter presupposes the existence of objectively coloured objects. Reference to the light sources having standard colours cannot be replaced by reference to them *looking* red, green, etc. The whole point is that there are no behavioural tests of whether a given standard really "looks the same" colour to different observers. Hence the objective colour of the standard source is needed precisely to give sense to second and third person application of the concept of something looking of that colour. Now if partial belief were similarly accessible only to introspection, lacking behavioural tests of its strength, Ramsey would equally have to rely on the standard coin actually being unbiased. It would not do for it merely to induce similar feelings of partial belief in it landing heads in different people. The whole point of the coin would be to give sense to such statements of similarity in feelings of partial belief. Ramsey's test would have to presuppose the existence of objectively equal chances of the coin landing heads and tails. Of this one would have to be convinced in any particular case by checks of symmetry, frequency, etc., just as one checks the specification of a standard source of red light. In each case one could be shown to be wrong, but only by reference to a further standard whose objective chance or colour was not in question.

This is of course a fantasy. Partial belief cannot possibly presuppose objective chance in the way coloured sensations may presuppose coloured objects. Perceiving objective colours is obviously a vastly more direct process than perceiving chances. I do not insist that the colour of sensations *is* thus definable in terms of their standard causes. But even if it were, that would give no reason to assume the same of partial belief. It is quite clear that our concept of partial belief could be applied just as well in a deterministic world devoid of chance. A subjectivist like de Finetti (1937) must indeed suppose this to be our situation. His view may be false but it is certainly not a contradiction in terms. On the other hand, Ramsey's is a patently reasonable device for comparing partial beliefs. Partial belief therefore cannot be a merely introspectible feeling. There must be independent public criteria for its presence and strength.

Partial belief may of course often be attended by related feelings; as