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0521615054 - Poisson Geometry, Deformation Quantisation and Group Representations

Edited by Simone Gutt, John Rawnsley and Daniel Sternheimer

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