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Edited by Simone Gutt, John Rawnsley and Daniel Sternheimer

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Poisson Geometry, Deformation Quantisation and Group Representations

Edited by

SIMONE GUTT

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Preface

This volume is devoted to the lecture courses given to the EuroSchool PQR2003 on “Poisson Geometry, Deformation Quantisation and Group Representations” held at the Université Libre de Bruxelles from the 13th to the 17th of June, 2003.

The EuroSchool was followed by a EuroConference from the 18th to the 22nd June with a large intersection of the audiences. These linked EuroSchool and EuroConference were made possible by the generous support of the European Commission through its High Level Scientific Conferences programme, contract HPCF-CT-2002-00180, the Fonds National belge de la Recherche Scientifique, the Communauté Française de Belgique, and the Université Libre de Bruxelles. The proceedings of the Euroconference appear separately in a volume of Letters in Mathematical Physics.

The idea to organise this meeting was born soon after the Conferences Moshe Flato in Dijon, in 1999 and 2000. We thought that Moshe would have appreciated having a new meeting centred around Poisson Geometry, Deformation Quantisation and Group Representations and which would incidentally celebrate important birthdays of four of our friends who have made fundamental contributions to this area: Boris Fedosov, Wilfried Schmid, Daniel Sternheimer, and Alan Weinstein. We particularly thank Alan, Daniel and Wilfried who accepted this proposal and helped with organising it.

There were four series of four one-hour lectures each given by Alberto Cattaneo on *Formality and Star Products*; Ieke Moerdijk and Janez Mrčun on *Lie Groupoids and Lie Algebroids*; Wilfried Schmid on *Geometric Methods in Representation Theory*; Alan Weinstein on *Morita Equivalence in Poisson Geometry*; and a two-hour broad presentation given by Daniel Sternheimer on *Deformation theory: A powerful tool in*

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physics modelling. We are extremely grateful to the lecturers and their co-authors for agreeing to write up and publish their lectures, to the London Mathematical Society for publishing them in their Lecture Notes series and to Cambridge University Press for their editorial support.

The editors wish to thank all those who took part in both meetings, with special thanks to the members of the scientific committee and to the lecturers. We also wish to thank all those who helped with the practical organisation, in particular Christine and Luc Lemaire, Monique Parker and Isabelle Renders.

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