

Automata Theory with Modern Applications

Recent applications to biomolecular science and DNA computing have created a new audience for automata theory and formal languages. This is the only introductory book to cover such applications. It begins with a clear and readily understood exposition of the basic principles, that assumes only a background in discrete mathematics. The first five chapters give a gentle but rigorous coverage of regular languages and Kleene's Theorem, minimal automata and syntactic monoids, Turing machines and decidability, and explain the relationship between context-free languages and pushdown automata. They include topics not found in other texts at this level, including codes, retracts, and semiretracts. The many examples and exercises help to develop the reader's insight. Chapter 6 introduces combinatorics on words and then uses it to describe a visually inspired approach to languages that is a fresh but accessible area of current research. The final chapter explains recently-developed language theory coming from developments in bioscience and DNA computing.

With over 350 exercises (for which solutions are available), plenty of examples and illustrations, this text will be welcomed by students as a contemporary introduction to this core subject; others, new to the field, will appreciate this account for self-learning.

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With contributions by Tom Head

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Preface

This book serves two purposes, the first is as a text and the second is for someone wishing to explore topics not found in other automata theory texts. It was originally written as a text book for anyone seeking to learn the basic theories of automata, languages, and Turing machines. In the first five chapters, the book presents the necessary basic material for the study of these theories. Examples of topics included are: regular languages and Kleene's Theorem; minimal automata and syntactic monoids; the relationship between context-free languages and pushdown automata; and Turing machines and decidability. The exposition is gentle but rigorous, with many examples and exercises (teachers using the book with their course may obtain a copy of the solution manual by sending an email to solutions@cambridge.org). It includes topics not found in other texts such as codes, retracts, and semiretracts.

Thanks primarily to Tom Head, the book has been expanded so that it should be of interest to people in mathematics, computer science, biology, and possibly other areas. Thus, the second purpose of the book is to provide material for someone already familiar with the basic topics mentioned above, but seeking to explore topics not found in other automata theory books.

The two final chapters introduce two programs of research not previously included in beginning expositions. Chapter 6 introduces a visually inspired approach to languages allowed by the unique representation of each word as a power of a primitive word. The required elements of the theory of combinatorics on words are included in the exposition of this chapter. This is an entirely fresh area of research problems that are accessible on the completion of Chapter 6. Chapter 7 introduces recently developed language theory that has been inspired by developments in the biomolecular sciences and DNA computing. Both of these final chapters are kept within automata theory through their concentration on results in regular languages. Research in progress has begun to extend these concepts to broader classes of languages. There are now specialized books on

DNA-computing – and in fact a rapidly growing Springer-Verlag Series on ‘Natural Computing’ is in progress. This book is the first one to link (introductory) automata theory into this thriving new area.

Readers with a strong background will probably already be familiar with the material in Chapter 1. Those seeking to learn the basic theory of automata, languages, and Turing machines will probably want to read the chapters in order. The sections on retracts and semiretracts, while providing interesting examples of regular languages, are not necessary for reading the remainder of the book.

A person already familiar with the basics of automata, languages, and Turing machines, will probably go directly to Chapters 6 and 7 and possibly the sections on retracts and semiretracts.

I thank Tom Head for the work he has done on this book including his contributions of Chapters 6 and 7 as well as other topics. I also thank Brett Bernstein for his excellent proofreading of an early version of the book and Kristin and Phil Muzik for creating the figures for the book. Finally I would like to thank Ken Blake and David Tranah at Cambridge University Press for their help and support.