Introduction

Global analysis has as its primary focus the interplay between the local analysis and the global geometry and topology of a manifold. This is seen classically in the Gauss–Bonnet theorem and its generalizations, which culminate in the Atiyah–Singer Index Theorem [Atiyah and Singer 1968a]. This places constraints on the solutions of elliptic systems of partial differential equations in terms of the Fredholm index of the associated elliptic operator and characteristic differential forms which are related to global topological properties of the manifold.

The Atiyah–Singer Index Theorem has been generalized in several directions, notably by Atiyah and Singer themselves [1971] to an index theorem for families. The typical setting here is given by a family of elliptic operators $P = \{P_b\}$ on the total space of a fibre bundle $F \to M \to B$, where P_b is defined on the Hilbert space $L^2(p^{-1}(b), \mathrm{dvol}(F))$. In this case there is an abstract index class $\mathrm{ind}(P) \in K^0(B)$. Once the problem is properly formulated it turns out that no further deep analytic information is needed in order to identify the class. These theorems and their equivariant counterparts have been enormously useful in topology, geometry, physics, and in representation theory.

A smooth manifold M^n with an integrable p-dimensional subbundle F of its tangent bundle TM may be partitioned into p-dimensional manifolds called *leaves* such that the restriction of F to the leaf is just the tangent bundle of the leaf. This structure is called a *foliation* of M. Locally a foliation has the form $\mathbb{R}^p \times N$, with leaves of the form $\mathbb{R}^p \times \{n\}$. Locally, then, a foliation is a fibre bundle. However the same leaf may pass through a given coordinate patch infinitely often. So globally the situation is much more complicated.

Foliations arise in the study of flows and dynamics, in group representations, automorphic forms, groups acting on spaces (continuously or even measurably), and in situations not easily modeled in classical algebraic topology. For instance, a diffeomorphism acting ergodically on a manifold M yields a one-dimensional foliation on $M \times_{\mathbb{Z}} \mathbb{R}$ with almost all leaves dense. The space of leaves of a foliation in these cases is not decent topologically (every point is dense in the example above) or even measure-theoretically (the space may not be a standard

Borel space). Foliations carry interesting differential operators, such as signature operators along the leaves. Following the Atiyah–Singer pattern, one might hope that there would be an analytic index class of the type

$$\mathrm{ind}\{P\} = \text{ Average } \mathrm{ind}(P_x).$$

There are two difficulties. First of all, leaves of compact foliations need not be compact, so an elliptic operator on a leaf may well have infinite-dimensional kernel or cokernel, and thus "$\mathrm{ind}(P_x)$" makes no sense. This problem aside, the fact that the space of leaves may not be even a standard Borel space suggests strongly that there is no way to average over it. There was thus no analytic index to try to compute for foliations.

Alain Connes saw his way through these difficulties. He realized that the "space of leaves" of a foliation should be a noncommutative space — that is, a C^*-algebra $C_r^*(G(M))$. In the case of a foliated fibre bundle this algebra is stably isomorphic to the algebra of continuous functions on the base space. This suggests $K_0(C_r^*(G(M)))$ as a home for an abstract index $\mathrm{ind}(P)$ for tangentially elliptic operators.

Next Connes realized that in the fibre bundle case there is an invariant transverse measure ν which corresponds to the volume measure on B. So we must assume given some invariant transverse measure in general. These may not exist. If one exists it may not be unique up to scale. An invariant transverse measure ν gives rise to a trace ϕ_ν on $C_r^*(G(M))$ and thus a real number

$$\mathrm{ind}_\nu(P) = \phi_\nu(\mathrm{ind}(P)) \in \mathbb{R}$$

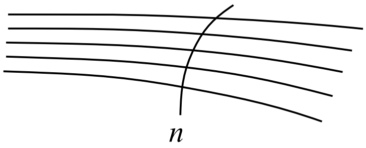
which Connes declared to be the *analytic index*. Actually we are cheating here; the most basic definition of the analytic index is in terms of locally traceable operators as we shall explain below and in Chapters I and IV. With an analytic index to compute, Connes computed it.

Connes Index Theorem. *Let M be a compact smooth manifold with an oriented foliation and let ν be an invariant transverse measure with associated Ruelle–Sullivan current C_ν. Let P be a tangentially elliptic pseudodifferential operator. Then*

$$\mathrm{ind}_\nu(P) = \big\langle \mathrm{ch}(P)\,\mathrm{Td}(M), [C_\nu] \big\rangle.$$

Connes' theorem is very satisfying. Its proof involves a tour of many areas of modern mathematics. We decided to write an exposition of this theorem and to use it as a centerpiece to discuss this region of mathematics. Along the way we realized that the setting of *foliated spaces* (local picture $\mathbb{R}^p \times N$ with N not necessarily Euclidean) was at once simpler pedagogically and yielded a somewhat more general theorem, since foliated spaces which are not manifolds occur with some frequency.

The local picture of a foliated space is a topological space of the form $L \times N$, where L is a copy of $\mathbb{R}^p$ and N is a separable metric space, not necessarily a manifold.



A *tangentially smooth* function

$$f : L \times N \to \mathbb{R}$$

is a continuous function with the following properties:

(1) For each $n \in N$, the function $f(\cdot, n) : L \to \mathbb{R}$ is smooth.

(2) All partial derivatives of f in the L directions are continuous on $L \times N$.

This notion extends naturally to tangentially smooth functions

$$f : L_1 \times N_1 \to L_2 \times N_2.$$

Definition. A *foliated space* X of dimension p is a separable metrizable space equipped with a regular foliated atlas $\mathcal{U} = \{(U_\alpha, \varphi_\alpha)\}$ such that, whenever U_α and U_β intersect, the composition

$$\varphi_\alpha(U_\alpha \cap U_\beta) \xrightarrow{\varphi_\alpha^{-1}} U_\alpha \cap U_\beta \xrightarrow{\varphi_\beta} \varphi_\beta(U_\alpha \cap U_\beta)$$

is tangentially smooth. A *tangentially smooth* function $f : X_1 \to X_2$ of foliated spaces is a continuous function such that if $U_i \subset X_i$ are foliated charts with associated maps

$$\varphi_i : U_i \longrightarrow L_i \times N_i$$

then the composition

$$\varphi_1(U_1 \cap f^{-1}(U_2)) \xrightarrow{\varphi_1^{-1}} U_1 \cap f^{-1}(U_2) \xrightarrow{f} f(U_1) \cap U_2 \xrightarrow{\varphi_2} L_2 \times N_2$$

is tangentially smooth.

This guarantees that the leaves in each coordinate patch coalesce to form leaves ℓ in X which are smooth p-manifolds, and that there is a natural vector bundle $FX \to X$ of dimension p which restricts to the tangent bundle of each leaf.

Any foliated manifold is a foliated space. There are interesting examples of foliated spaces which are not foliated manifolds. For instance, a solenoid is a foliated space with leaves of dimension 1 and with N_i homeomorphic to Cantor sets. If M^n is a manifold which is foliated by leaves of dimension p and if N is a transversal of M^n then any subset of N determines a foliated subspace of M simply by taking those leaves of M^n which meet the subset. This includes the laminations of much current interest in low-dimensional topology, and it includes the topological spaces that arise in the study of quasicrystals and tilings of Euclidean space. Finally, X may well be infinite-dimensional: take $\prod_1^\infty S^1$ foliated by lines corresponding to algebraically independent irrational rotations. Then $\{1\} \times \prod_2^\infty S^1$ is transversal![1]

If $E \xrightarrow{\pi} X$ is a foliated bundle (i.e., E is also foliated, π takes leaves to leaves, and π is tangentially smooth) then $\Gamma_\tau(E) \equiv \Gamma_\tau(X, E)$ denotes continuous tangentially smooth sections of E. We let

$$\Omega_\tau^k(X) = \Gamma_\tau(\Lambda^k F^*)$$

and define the *tangential cohomology* groups of a foliated space by

$$H_\tau^k(X) = H^k(\Omega_\tau^*(X)),$$

where $d : \Omega_\tau^k(X) \to \Omega_\tau^{k+1}(X)$ is the analogue of the de Rham differential obtained by differentiating in the leaf directions. Similar (but not the same) groups have been studied by many authors. Tangential cohomology groups are based upon forms which are *continuous* transversely (even if X is a foliated manifold.) It turns out that this small point has some major consequences. The groups may be described as

$$H_\tau^k(X) = H^k(X : \mathcal{R}_\tau),$$

where $\mathcal{R}_\tau$ is the sheaf of germs of continuous functions which are constant along leaves. The tangential cohomology groups are functors from foliated spaces and leaf-preserving tangentially smooth maps to graded commutative $\mathbb{R}$-algebras. They vanish for $k > p$. There is the usual apparatus of long exact sequences, suspension isomorphisms, and a Thom isomorphism for oriented k-plane bundles.

The groups $H_\tau^*(X)$ have a natural topology and are not necessarily Hausdorff; we let

$$\overline{H}_\tau^k(X) = H_\tau^k(X)/\overline{\{0\}}$$

denote the maximal Hausdorff quotient. For example, if X is the torus $S^1 \times S^1$ foliated by an irrational flow, $H_\tau^1(X)$ has infinite dimension but $\overline{H}_\tau^1(X) \cong \mathbb{R}$. The parallel between de Rham theory and tangential cohomology theory extends to the existence of characteristic classes. Given a tangentially smooth vector bundle $E \to X$ we construct tangential connections, curvature forms, and Chern classes.

Next we recall the construction of the groupoid of a foliated space; the idea is due to Ehresmann, Thom and Reeb and was elaborated upon by Winkelnkemper. A foliated space X has a natural equivalence relation: $x \sim y$ if and only if x

[1] This paragraph appeared in the first edition. Since then there has been an explosion of interest in laminations. (Nowadays some authors use the word *lamination* as a synonym for *foliated space*.) The books [Candel and Conlon 2000; 2003] contain a host of examples and references.

and y are on the same leaf. The resulting space $\mathcal{R}(X) \subset X \times X$ is not a well-behaved topological space. The holonomy groupoid $G(X)$ of a foliated space is designed to by-pass this difficulty. It contains holonomy data not given by $\mathcal{R}(X)$; holonomy is essential for diffeomorphism and structural questions about the foliated space. The *holonomy groupoid* $G(X)$ consists of triples $(x, y, [\alpha])$, where x and y lie on the same leaf ℓ of X, α is a path from x to y in ℓ, and $[\alpha]$ denotes the holonomy class of the path α. The map $G(X) \to \mathcal{R}(X)$ is simply $(x, y, [\alpha]) \to (x, y)$. The preimages of (x, y) correspond to holonomy classes of maps from x to y. The space $G(X)$ is a (possibly non-Hausdorff) foliated space. If N is a complete transversal (meaning that N is Borel and for each leaf ℓ the intersection $N \cap \ell$ is nonempty and at most countable), G_N^N is the subgroupoid of $G(X)$ consisting of triples $(x, y, [\alpha])$ with $x, y \in N$. In a sense which we make precise, G_N^N is a good discrete model for $G(X)$.

We turn next to a study of differential and pseudodifferential operators on X. Suppose that E_0 and E_1 are foliated bundles over X and

$$D : \Gamma_\tau(E_0) \to \Gamma_\tau(E_1).$$

The operator D is said to be *tangential* if D restricts to

$$D_\ell : \Gamma(E_0 \,|_\ell) \to \Gamma(E_1 \,|_\ell)$$

for each ℓ, and D is *tangentially elliptic* if each operator D_ℓ is an elliptic operator. If D is a tangential, tangentially elliptic operator then Ker D_ℓ and Ker D_ℓ^* consist of smooth functions on l. These spaces may well be infinite-dimensional, and hence expressions such as

$$\dim \operatorname{Ker} D_\ell - \dim \operatorname{Ker} D_\ell^*$$

make no sense. However there is some additional structure at our disposal, for Ker D_ℓ and Ker D_ℓ^* are $C^\infty(\ell)$-modules. We shall show that these spaces are for each ℓ *locally* finite-dimensional in a sense that we now describe.

Let Y be a locally compact space endowed with a measure (in the application to index theory $Y = \ell$ is a leaf and the measure is a volume measure) and suppose that T is a positive operator on $L^2(Y, E)$ for some bundle E over Y. Then

$$\operatorname{Trace}(f^{1/2} T f^{1/2}) = \operatorname{Trace}(T^{1/2} f T^{1/2})$$

for every bounded positive function f. We define a measure μ_T by

$$\operatorname{Trace}(f^{1/2} T F^{1/2}) = \int_Y f \, d\mu_T$$

and declare T to be *locally traceable* with local trace μ_T if, for some family $\{Y_i\}$ of compact sets with union Y, we have

$$\mu_T(Y_i) < \infty.$$

If

$$T = \sum \lambda_i T_i$$

with each T_i locally traceable, T is locally traceable with local trace $\mu_T = \sum \lambda_i \mu_{T_i}$. We identify a closed subspace $V \subset L^2(Y, E)$ with the orthogonal projection onto it and say that the subspace is *locally finite-dimensional* if the projection is locally traceable. Any closed subspace of $L^2(Y, E)$ that consists entirely of continuous functions is easily seen to be locally finite-dimensional.

If Y is a C^∞ manifold and D is an elliptic pseudodifferential operator on Y then DD^* and D^*D are locally traceable so Ker D and Ker D^* are locally finite-dimensional. The *local index* of D is defined to be

$$\iota_D = \mu_{\text{Ker}\, D} - \mu_{\text{Ker}\, D^*}.$$

If Y is a compact manifold then $\int_Y \iota_D = \text{ind}(D)$, the classical Fredholm index.

The notion of locally traceable operator makes it possible to discuss the index of an elliptic operator on a noncompact manifold. As we observed previously, if D is a tangential, tangentially elliptic operator on a compact foliated space X then D_ℓ is an elliptic operator on the leaf ℓ and its local index

$$\iota_{D_\ell} = \mu_{\text{Ker}\, D_\ell} - \mu_{\text{Ker}\, D_\ell^*}$$

does make sense as a (signed) Radon measure on ℓ. Write $\iota_D^x = \iota_{D_\ell}$ for each $x \in \ell$. Then $\iota_D = \{\iota_D^x\}$ is a *tangential measure*; that is, a family of Radon measures supported on leaves of X with suitable invariance properties (see Definition 4.11). We regard ι_D as the abstract analytic index of D. If the foliation bundle F is oriented then a tangential measure determines a class in $\overline{H}_\tau^p(X)$. The task of an index theorem is to identify that class.

To proceed further along these lines and as they are of substantial independent interest, we introduce transverse measures. For this we move temporarily to a measure-theoretic context. Suppose that $(X, \mathcal{R})$ is a standard Borel equivalence relation. We assume that there is a complete Borel transversal; that is, a Borel set which meets all equivalences classes and where the intersection with each class is denumerable. This condition holds easily in the setting of foliated spaces. Assume further that we are given a one-cocycle $\Delta \in Z^1(\mathcal{R}, \mathbb{R}^*)$. A *transverse measure* of modulus Δ is a measure ν on the σ-ring of all Borel transversals which is σ-finite on each transversal and such that $\nu\,|_T$ is quasi-invariant with modulus $\Delta\,|_T$ for the countable equivalence relation $\mathcal{R} \cap (T \times T)$ for each transversal T. If $\Delta \equiv 1$ then ν is an *invariant* transverse measure. For example, if X is the total space of a fibration $\ell \to X \to B$ foliated with fibres as leaves then an invariant transverse measure on X is precisely a σ-finite measure on B.

Recall that a tangential measure λ is an assignment $\ell \mapsto \lambda_\ell$ of a measure to each leaf (or class of $\mathcal{R}$) which satisfies suitable Borel smoothness properties (see Definition 4.11). For example, if D is a tangential, tangentially elliptic operator on X then the local index ι_D is a tangential measure. If we choose a coherent family of volume measures for each leaf ℓ then these coalesce to a tangential measure.

Given a tangential measure λ and an invariant transverse measure ν, we describe an integration process that produces a measure $\lambda\, d\nu$ on X and a number $\int \lambda\, d\nu$ obtained by taking the total mass of the measure. Choose a complete transversal N. There is a Borel map $\sigma : X \to N$ with $\sigma(x) \sim x$. Regard X as fibring measure-theoretically over N, and let λ_n be the restriction of λ_ℓ to the set $\sigma^{-1}(n)$, which is contained in the leaf where n lies. Then $\int_N \lambda_n\, d\nu(n) = \lambda\, d\nu$ is a measure on X. This integration process is related to the pairing of currents with foliation cycles in [Sullivan 1976].

How many invariant transverse measures are there? Let $MT(X)$ be the vector space of Radon invariant transverse measures. The construction above provides a pairing

$$MT(X) \times \Omega_\tau^p(X) \to \mathbb{R}$$

and hence a Ruelle–Sullivan map

$$MT(X) \to \mathrm{Hom}_{\mathrm{cont}}(H_\tau^p(X), \mathbb{R}) \cong H_p^\tau(X).$$

We prove a Riesz representation theorem: this map is an isomorphism. For example, if X is foliated by points then $H_\tau^0(X) = C(X)$ and an invariant transverse measure is just a measure, so our result reduces to the usual Riesz representation theorem. We see also that X has no invariant transverse measure if and only if $\bar{H}_\tau^p(X) = 0$.

With this machinery in hand we can state and prove the remarkable index theorem of A. Connes. Let D be a tangential, tangentially elliptic pseudodifferential operator on a compact oriented foliated space of leaf dimension p. As described above, we obtain the analytic index of D as a tangential measure ι_D. For any invariant transverse measure ν the real number $\int_X \iota_D\, d\nu$ is the analytic ν-index $\mathrm{ind}_\nu(D)$ defined by Connes. The Connes index theorem states that for any invariant transverse measure ν,

$$\int \iota_D\, d\nu = \int \iota_D^{\mathrm{top}}\, d\nu,$$

where

$$\iota_D^{\mathrm{top}} = \pm\big[\Phi_\tau^{-1}\, \mathrm{ch}_\tau(D)\big]\, \mathrm{Td}_\tau(X)$$

is the topological index of the symbol of D. (See Chapter V for definitions). Using the Riesz representation theorem we reformulate Connes' theorem to read

$$[\iota_D] = [\iota_D^{\mathrm{top}}] \in \bar{H}_\tau^p(X)$$

which, as is evident, does not involve invariant transverse measures. Of course if X has no invariant transverse measures then $\bar{H}_\tau^p(X) = 0$ and $\iota_D \in \overline{\{0\}}$.

There is a stronger form of the index theorem for foliated manifolds which is due to Connes and Skandalis. To state it we need to introduce the reduced C^*-algebra of the foliated space. The compactly supported tangentially smooth functions on $G(X)$ form a $*$-algebra under convolution. (If $G(X)$ is not Hausdorff then a modification is required.) For each leaf G^x of $G(X)$ with its natural volume measure there is a natural regular representation of this $*$-algebra on $\mathcal{B}(L^2(G^x))$. Complete the $*$-algebra with respect to these representations and one obtains $C_r^*(G(X))$. This algebra enters into index theory because there is a natural pseudodifferential operator extension

$$0 \to C_r^*(G(X)) \longrightarrow \bar{\mathcal{P}}^0 \xrightarrow{\sigma} \Gamma(S^*F,\ \mathrm{End}(E)) \to 0$$

and hence the tangential principal symbol of D yields an element of

$$K_0(C_r^*(G(X))).$$

Connes and Skandalis [Connes and Skandalis 1984] identify this element and thereby obtain a sharper form of the index theorem which is useful in the Type III situation. Even in the presence of an invariant transverse measure, if the symbol of an operator D has finite order in $K_0(C_r^*(X))$ then $[\iota_D] = 0$ in $H_\tau^p(X)$.

We conclude this introduction with a brief summary of the contents of each chapter.

I. Locally Traceable Operators

Given an operator T on $L^2(Y, E)$ for a locally compact space Y, we explain the concept of local traceability and we construct the local trace μ_T of T. The local index ι_D of an elliptic operator on a noncompact manifold is one motivating example. We also discuss several situations outside the realm of foliations where locally traceable operators shed some light. In particular, we interpret the formal degree of a representation of a unimodular locally compact group in these terms.

II. Foliated Spaces

Here we set forth the topological foundations of our study. We give many examples of foliated spaces and construct tangentially smooth partitions of unity. Then follow smoothing results which enable us, for instance, to assume freely that bundles over our spaces are tangentially smooth. It is perhaps worth noting that $K^0(X)$ coincides with the subgroup generated by tangentially smooth bundles. Next we explain holonomy and, following Winkelnkemper, introduce the holonomy groupoid of a foliated space. We consider the relationship between $G(X)$ and its discrete model G_N^N and determine the structure of G_N^N in several examples.

III. Tangential Cohomology

In this chapter we define the tangential cohomology groups $H_\tau^*(X)$ as the cohomology of the de Rham complex $\Gamma_\tau(\Lambda^* F^*)$ and equivalently as the cohomology of X with coefficients in the sheaf of germs of continuous functions on X which are constant along leaves. There is an analogous compactly supported theory $H_{\tau c}^*(X)$ and an analogous tangential vertical theory $H_{\tau v}^*(E)$ on bundles. We develop the properties parallel to the expected properties from de Rham theory. There is a Mayer–Vietoris sequence (for open subsets) and a Künneth isomorphism

$$H_\tau^*(X) \otimes H^*(M) \xrightarrow{\cong} H_\tau^*(X \times M)$$

provided that M is a manifold foliated as one leaf and $X \times M$ is foliated with leaves $\ell \times M$. We establish a Thom isomorphism theorem (3.30) of the type

$$\Phi : H_\tau^k(X) \xrightarrow{\cong} H_{\tau v}^{n+k}(E)$$

for an oriented tangentially smooth n-plane bundle $E \to X$. Finally we indicate the definition of tangential homology theory. In an appendix we rephrase these constructions in terms of Lie algebra cohomology.

IV. Transverse Measures

We develop here the general theory of groupoids, both in the measurable and topological contexts, in order to give a proper home to transverse measures. The prime examples are $G(X)$ and G_N^N, of course. We introduce transverse measures and their elementary properties. The proper integrands for transverse measures are tangential measures, as we have previously explained in the foliation context. We carefully explain the integration process

$$(\lambda, \nu) \mapsto \lambda \, d\nu \mapsto \int \lambda \, d\nu$$

and indicate the necessary boundedness results. Specializing to topological groupoids and continuous Radon tangential measures, we recount the Ruelle–Sullivan construction of the current $C_\nu \in \Omega_p^\tau(X)$ associated to the transverse measure ν. The current is a cycle if and only if ν is an invariant transverse measure.

Next we relate the space of invariant transverse measures $MT(X)$ on X to invariant measures on a complete transversal N. Finally we establish the Riesz representation theorem: if X is a compact oriented foliated space with leaf dimension p then the Ruelle–Sullivan map

$$MT(X) \longrightarrow \mathrm{Hom}_{\mathrm{cont}}(H_\tau^p(X), \mathbb{R})$$

is an isomorphism. One useful consequence of this result is that a linear functional F on $MT(X)$ is representable as $F(\nu) = \int \omega \, d\nu$ for some $\omega \in H_\tau^p(X)$ if and only if the functional is continuous in the weak topology on $MT(X)$.

V. Characteristic Classes

This chapter contains the Chern–Weil development of tangential characteristic classes. This comes down to carefully generalizing the usual constructions of connections, curvature, and their classes. This results in tangential Chern classes $c_n^\tau \in H_\tau^{2n}(X)$, tangential Pontryagin classes $p_n^\tau \in H_\tau^{4n}(X)$, and a tangential Euler class, as well as the now classical universal combinations of these. We construct these classes at the level of forms, so that, for a fixed tangential Riemannian connection, the topological index is a uniquely defined form. We verify the necessary properties of the tangential Chern character and the tangential Todd genus which relates the K-theory and tangential cohomology Thom isomorphisms.

VI. Operator Algebras

Each foliated space has an associated C^*-algebra $C_r^*(G(X))$ introduced by A. Connes. In this chapter we present its basic properties. Central to our treatment is the Hilsum–Skandalis isomorphism

$$C_r^*(G(X)) \cong C_r^*(G_N^N) \otimes \mathcal{K},$$

which shows that, at the level of C^*-algebras, the foliated space “fibres” over a complete transversal N. The C^*-algebra $C_r^*(G_N^N)$ is the C^*-algebra of the discrete model G_N^N of $G(X)$. An invariant transverse measure ν induces a trace ϕ_ν on $C_r^*(G(X))$, and one then may construct the von Neumann algebra $W^*(G(X), \tilde{\mu})$. The analogous splitting

$$W^*(G(X), \tilde{\mu}) \cong W^*(G_N^N, \tilde{\mu}) \otimes \mathcal{B}(\mathcal{H})$$

at the von Neumann algebra level *is* expected, of course. In the ergodic setting this corresponds to the usual decomposition of a II_∞ factor into the tensor product of II_1 and I_∞ factors. We conclude with a brief introduction to K-theory and the construction of a partial Chern character $c : K_0(C_r^*(G)) \to \overline{H}_\tau^p(X)$.

VII. Pseudodifferential Operators

The usual theory of pseudodifferential operators takes place on a smooth manifold. In this chapter we “parametrize” the theory to the setting of foliated spaces. This involves constructing the pseudodifferential operator algebra and its closure, defining the tangential principal symbol, and showing that the analytic