

Index

When a term is encountered on numerous occasions, the page number of only three of its appearances are given; the number in italics indicates the page where the term was defined.

- absorbing state, *18, 200, 361*
- absorption probability, *27*
- aggregated likelihood, *435, 436, 438*
- algebraic multiplicity, *108, 110, 193*
- almost sure (AS) convergence (see also: convergence with probability 1), *61, 225, 370*
- alternative (hypothesis), *350, 351, 422*
 - composite, *351*
 - simple, *350*
- aperiodic communicating class, *24*
- aperiodic Markov chain, *24, 71, 402*
- arrival process, *291, 300, 318*
- arrival rate, *292, 298, 304*

- backward equations (Kolmogorov backward equations), *187, 242, 316*
- Baum–Welch learning algorithm, *443*
- Baum–Welch transformation, *443*
 - in an interpolation HMM problem, *445, 450*
 - in a filtration HMM problem, *459, 473*
- bipartite graph, *118, 133*
- birth process (BP), *240, 241, 251*
- birth-and-death process (BDP), *21, 248, 341*
- birth-and-death Markov chain, *53, 86*
- branching process, *305, 306*
- Burke’s theorem, *296, 302, 341*
- busy period (of a queue), *295, 335*

- Cauchy–Schwarz (CS) inequality, *125, 136, 465*
- Central limit theorem, *139, 143*
- characteristic equation, *8, 25, 193*
- characteristic polynomial, *11, 110, 360*
- Chebyshev inequality, *139, 367, 378*
- Cheeger’s inequality, *120, 121, 135*
- Chernoff inequality, *142*
- closed communicating class, *18, 277, 355*
- communicating class, *18, 161, 260*
- connected graph, *83, 84, 184*
- consistent estimator, *366, 367, 450*

- continuous-time Markov chain (CTMC), *196, 197, 341*
- controlled Markov chain, *89*
- convergence in distribution, *229*
- convergence in probability, *367, 373, 382*
- convergence to equilibrium, *70, 122, 284*
 - geometric (exponential), *74, 99*
- convergence with probability 1 (see also: almost sure(AS) convergence), *61, 196, 370*
- convex order, *433*
- correlation coefficient, *359*
- coupling (of Markov chains), *72, 257, 431*
- covariance, *359, 407, 471*
- Cramér’s theorem, *144, 146, 148*
- CTMC, (δ, Q) , *197*
- CTMC, (λ, Q) , *196, 256, 286*
- cycle, *119, 180, 310*
 - directed, *119, 126, 172*
 - server’s, *295*

- departure process, *291, 298, 341*
- detailed balance equations (DBEs), *82, 286, 304*
- determinant of a matrix, *99, 108, 110, 194*
- directed cycle, *119, 126, 172*
- Dirichlet probability density function (Dirichlet PDF), *404, 405, 410*
- Dirichlet distribution, *401, 406, 408*
- Dirichlet form, *121, 124,*
- Dirichlet integral formula, *403, 407, 409*
- discrete Fourier transform, *101*
- discrete Laplacian, *107, 130, 131*
- discrete-time Markov chain (DTMC), *6, 8, 446*

- Ehrenfest urn problem, *107*
- eigenbasis, *115*
- eigenspace, *110, 112, 149*
- eigenvalue, *viii, 8, 471*
- eigenvector, *8, 11, 395*
- embedded h -spacing, *271*
- embedded jump chain, *257*

empirical distribution, 422
 empirical frequency, 445, 446
 entropy rate, 389
 epidemics, 345
 ergodic, 62, 76
 Erlang's formulas, 302
 equilibrium distribution (ED), viii, 52, 198
 estimation, 349, 390, 468
 estimator, 350, 357, 456
 expectation–modification (EM) algorithm, 462, 466, 471
 explosion, 245, 251, 256
 explosion time, 269, 331
 explosive birth process, 247, 254,
 explosive Markov chain, 230, 244, 255
 explosive state, 256
 exponential family, 466, 468

filtration problem for HMM, 438, 439, 459
 first passage time, 35, 181
 Fisher information, 419, 420
 forward equations (Kolmogorov forward equations), 187, 242, 334
 full likelihood (for a Markov chain sample), 350, 358, 361

G/M/1 queue, 343
 Gärtner–Ellis theorem, 148
 generalised expectation–modification (GEM) algorithm, 462, 471
 generator, 196, 200, 334
 geometric distribution, 306
 geometric multiplicity, 109, 110, 193
 Gibbs' inequality, 421, 426, 471
 global convergence theorem, for EM algorithms, 477
 graph, 28, 184, 309

Hamiltonian, 127
 Hermitian matrix, 85, 162, 355
 hidden Markov model (HMM), 434
 filtration problem, 437, 438, 478
 interpolation problem, 437, 444, 474
 hitting probability, 27, 171, 266
 hitting time, 26, 199, 266
 holding time, 180, 202, 314
 homogeneous function, 447, 482
 homogeneous Markov chain, 17, 67, 201
 homogeneous Poisson process, 233, 236
 homogeneous random walk, 45, 248, 249
 hypothesis testing, 350, 422

idempotent matrix, 8, 395
 independent, identically distributed (IID), 7, 154, 469
 independent increments, 211, 232, 322
 inhomogeneous Markov chain, 17, 201, 234
 inhomogeneous Poisson process (IPP), 231, 254, 336
 initial distribution, 3, 6, 444
 inspection paradox, 220, 227, 229
 invariant measure (IM), 53, 259, 322

invariant distribution, 52, 283, 382
 irreducible, 22, 43, 402
 Ising model, 127, 129, 130 jump chain, 39, 257, 345

jump rate, 185, 255, 345

Kantorovich inequality, 470, 471
 Key Renewal theorem, 343
 Kullback divergence, 420, 429, 431
 conditional, 430
 Kullback–Leibler distance, 420, 426, 432

Laplacian, 107, 130, 131
 large deviation principle (LDP), 148, 150, 152
 large deviation rate (LDR) function, 148, 151
 large deviation theory, 138, 143
 Law of Large numbers (LLN), 61, 481
 Lebesgue measure, 355, 356, 371
 Lebesgue's theorem (dominated convergence), 226
 Legendre transform (Legendre–Fenchel, Legendre–Cramér transform), 140, 142, 143
 likelihood, 350, 436, 475
 likelihood ratio, 350, 351, 387
 monotone, 432
 Liouville distribution, 401, 402, 405
 Liouville formula, 405
 log-likelihood, 358, 462, 472
 log-likelihood ratio, 422, 432
 long-run proportion, 76, 169, 314
 loss model, 299, 301, 341
 Luzin theorem, 371
 Lyapunov function, 473
 Lyapunov theorem, 475

M/GI/1 queue, 342,
 M/M/1 chain, 293, 296, 302
 M/M/1/∞ queue, 292, 295, 304
 M/M/r/∞ queue, 293, 299
 M/M/∞ chain, 298, 299
 M/M/∞ queue, 293, 297, 298
 M/M/r/c chain, 299, 300, 302
 Markov chain, 6, 80, 122
 Markov property, 13, 198, 455
 maximum likelihood equation, 361, 366, 377
 maximum likelihood estimator (MLE), 358, 376, 436
 minimal chain, 254, 259, 329
 minimal non-negative solution, 27, 266, 324
 moment-generating function (MGF), 139, 246, 339

nearest-neighbour random walk, 45, 107, 322
 Neyman–Pearson lemma, 350, 351
 non-closed communicating class, 18, 23, 43
 non-essential state, 18, 22, 25
 non-explosive birth process, 247
 non-explosive birth-and-death process, 247
 non-explosive Markov chain, 230, 244, 255
 null hypothesis, simple, 350, 351, 351, 422
 null recurrent (NR) Markov chain, 57, 273, 341
 null recurrent (NR) state, 42, 59, 273
 null recurrent (NR) transition matrix or Q-matrix, 53, 57, 273

- open communicating class, 53, 80, 277
Ostrowski's theorem, 475
- partially observed Markov chain, 96
path (on a graph), 28, 47, 132
passage time, 35, 44, 200
period of a subclass, 24, 109, 355
periodic Markov chain, 100, 133, 355
periodic subclass, 24, 105, 284
periodic transition matrix, 109, 118, 133
permanent, 99
Perron–Frobenius theorem, 112, 149
Pochhammer symbol, 411
Poincaré bound, 118, 131, 134
Poincaré inequality, 118, 124, 131
Poisson arrival process, 318
Poisson distribution, 168, 298, 468
Poisson process, $PP(\lambda)$, 210, 240, 341
positive definite matrix, 424, 464, 470
positive recurrent (PR) state, 42, 59, 72
positive recurrent (PR) Markov chain, 57, 59, 273
positive recurrent (PR) state, 42, 59, 273
positive recurrent (PR) transition or Q-matrix, 54, 57, 273
probability density function (PDF), 238, 371, 433
probability distribution, 1, 61, 438
probability-generating function, 38, 139, 339
probability mass function (PMF), 2, 350, 433
probability measure, 1
projection Markov chain, 122, 124, 128
projection onto a subspace, 395
projection of a random walk, 52
- Q-matrix, 185, 248, 321
queue length (size), 291, 292, 341
queueing models, 291
queueing theory, 291
- random variable (RV) 61, 220, 446
random walk (RW) on a graph, 83, 184, 310
random walk (RW) on a lattice, 45, 163, 347
rate of convergence to equilibrium, 74, 122, 469
rate of a Poisson process, 163, 210, 341
rate of exponential distribution, 200, 202
record process (process of records), 237, 338
record values (records), 236, 237, 336
recording states of a Markov chain, 45, 96, 452
recurrent Markov chain, 44, 272, 322
recurrent communicating class, 43, 44
recurrent (R) state, 39, 269, 322
recurrent (R) transition matrix (Q-matrix), 44, 53, 56
reduced likelihood (for a Markov chain sample), 350, 358, 361
relative entropy, 144, 145, 420
renewal process, 301, 343
renewal time, 344
residual holding time, 200
restriction Markov chain, 123, 126, 128
return probability, 15, 69, 319
return time, 40, 162, 342
- reversible Markov chain, 82, 285, 341
reversible transition matrix, 114, 118, 135
- sample (of a Markov chain), 151, 366, 469
sample path (of a Markov chain), 66, 196, 319
sample trajectory (of a Markov chain), 45, 196, 392
scalar product, 87, 131, 288
score (of a random variable), 418, 419
second moment, 62, 398
Secretary problem, 93, 94
self-adjoint matrix, 87, 288
semigroup property, 187, 247, 335
simple random walk, 50, 164, 230
simplex (of stochastic vectors), 146, 151, 353
sojourn time (in a queue), 295
spectral circle, 113
spectral gap, 105, 126, 149
spectral radius, 113, 114, 149
spectrum of a matrix, 110, 118, 130
state space, I , 185, 445
Stirling's formula, 47, 66, 147
stochastic matrix, 2, 361, 438
stochastic order, 229
stochastic vector, 2, 146, 446
stochastically equivalent, 291, 298, 301
stochastically larger, 229
stochastically smaller, 229
stopping time, 35, 199, 220
strong Law of Large numbers (LLN), 61, 170, 389
strong Markov property, 35, 199, 268
sum of independent Poisson processes, 222
Sylvester's theorem, 394
symmetric matrix, 108, 162, 434
symmetric random walk, 46, 230, 322
- time reversal, 80, 87, 288
total variation distance, 122
training sequence, 437, 450, 473
transient communicating class, 43, 44
transient (T) Markov chain, 44, 269, 341
transient (T) state, 40, 59, 269
transient (T) transition matrix or Q-matrix, 44, 59, 269
transition count, 390, 411, 441
transition matrix, viii, 2, 451
transition probabilities, 3, 180, 480
transition probability matrix, 46, 209, 351
transition rate, 230, 250, 345
- unbiased estimator, 357, 358, 450
- valency, 83, 182, 310
- waiting time, 295
weak Law of Large numbers (LLN), 367, 368, 370
Whittle distribution, 393, 397, 401
Whittle's formula, 390, 393
Wilks' theorem, 352
- Yegorov theorem, 372
Yule–Furry process, 245