Probability and Statistics by Example: II

Probability and statistics are as much about intuition and problem solving, as they are about theorem proving. Because of this, students can find it very difficult to make a successful transition from lectures to examinations to practice, since the problems involved can vary so much in nature. Since the subject is critical in many modern applications such as mathematical finance, quantitative management, telecommunications, signal processing, bioinformatics, as well as traditional ones such as insurance, social science and engineering, the authors have rectified deficiencies in traditional lecture-based methods by collecting together a wealth of exercises for which they have supplied complete solutions. These solutions are adapted to needs and skills of students.

Following on from the success of *Probability and Statistics by Example: Basic Probability and Statistics*, the authors here concentrate on random processes, particularly Markov processes, emphasising models rather than general constructions. Basic mathematical facts are supplied as and when they are needed and historical information is sprinkled throughout.

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Probability and Statistics by Example: II
Markov Chains: a Primer in Random Processes and their Applications

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Preface

This volume, like its predecessor, *Probability and Statistics by Example*, Vol. 1, was initially conceived with the intention of giving Cambridge students an opportunity to check their level of preparation for Mathematical Tripos examinations. And, as with the first volume, in the course of the preparation, another goal became important: to give the general public a clearer picture of how probability- and statistics-related courses are taught in a place like the University of Cambridge, and what level of knowledge is achieved (or aimed for) by the end of these courses. In addition, the specific topic of this volume, Markov chains and their applications, has in recent years undergone a real surge. A number of remarkable theoretical results were obtained in this field which only twenty years or so ago was considered by many probabilists as a 'dead' zone. Even more surprisingly, an active part in this exciting development was played by applied research. Motivated by a dramatically increasing number of problems emerging in such diverse areas as computer science, biology and finance, applied people boldly invaded the territory traditionally reserved for the few hardened enthusiasts who until then had continued to improve old results by removing one or another condition in theorems which became increasingly difficult to read, let alone apply. We thus felt compelled to include some of these relatively recent ideas in our book, although the corresponding sections have little to do with current Cambridge courses. However, we have tried to follow a distinctively Cambridge approach (as we see it) throughout the whole volume.

On the whole, our feeling is that the modern theory of Markov chains can be compared with a huge and complex living organism which has suddenly woken from a period of hibernation and is now in a state of active consumption and digestion of fresh foodstuff produced by fertile lands around it, flourishing under blissful conditions. As often happens in nature, some parts of this living organism go through vast changes: they become less or more important compared with the previous state. In addition, some parts, like an old skin, may be sloughed off and replaced by new, better adapted to new realities. Our book then can be compared with a photographic snapshot of this giant from a certain distance and angle. We are not able to feature the whole animal (it is simply too big and fast-moving for us), and many details of the picture within the frame of our snapshot are blurred. However, we hope that the overall image is somewhat new and fresh.

At the same time, our goal was to treat those topics that are particularly important, especially in the course of learning the basic concepts of Markov chains. These are the aspects and issues that are particularly thought-provoking for a newcomer and, not surprisingly, usually provide the most fertile grounds for setting up problems suitable for exams. Roughly speaking, all the material from the theory of Markov chains which proved to be useful in examinations in Cambridge during the period 1991–2003 is included in the book.

It has to be said that studying via (or supporting the learning process by going through) a large number of homogeneous problems (with or without solutions) can be rather painstaking. A view popular among the mathematically-minded section of the academic community could be that the most productive way of learning a mathematical subject is to digest proofs of a collection of theorems general enough to serve many particular cases and then treat various questions as examples illustrating such theorems (the present authors were educated in precisely this fashion). The problem is that it ideally suits the mathematically-minded section of the academic community, but perhaps not the rest . . .

On the other hand, an increasing number of students (mainly, but not always, with a non-mathematical background) strongly oppose (at least psychologically) any attempt at a ‘decent’ proof of even basic theorems. Moreover, the manual calculations often required in examples whose tailor-made background is obvious also became increasingly unpopular with generations of students for whom computers have become as ordinary as toothbrushes. The authors can refer to their own experience as lecturers when audiences have been convinced more by computer evidence than by a formal proof. There is clearly a problem about how to teach an originally pure mathematics subject to a wider audience. There is some basis for the above unpopularity, although we personally still believe that learning the proof of convergence to an equilibrium distribution of a Markov chain is more productive than seeing twenty or so numerical examples confirming this fact. But an artificial example where, say, a four by four transition matrix is constructed so that its eigenvalues are of a ‘nice’ form (a particular value 1, easy to find from symmetry or an other ‘educated guess’, and the remaining two from a quadratic equation), may mis- or even back-fire, whereas an efficient modern package could do the job without much fuss. However, our presentation disregards these aspects; we consider it as first step towards a future style of book-writing.

A particular feature of the book is the presence of what we have called 'Worked Examples', along with 'Examples'. The former show readers how to go about solving specific problems; in other words, give explicit guidance about how to make the transition from the theory to the practical issue of solving problems. The end of a worked example is marked by a symbol □. The latter are illustrative, and are intended to reveal more about the underlying ideas. We must note that we have been particularly influenced by books Norris, 1997, and Stroock, 2005. In addition, a number of past and present members of the Statistical Laboratory, DPMMS, University of Cambridge, contributed to creating a particular style of presentation (we wrote about it in the preface to the first volume). It is the pleasure to name here David Williams, Frank Kelly, Geoffrey Grimmett, Douglas Kennedy, James Norris, Gareth Roberts and Colin Sparrow whose lectures we attended, whose lecture notes we read and whose example sheets we worked on. In Swansea, great help and encouragement came from Alan Hawkes, Aubrey Truman and again David Williams. We are particularly grateful to Elie Bassouls who read the early version of the book and made numerous suggestions for improving the presentation. His help extended beyond the usual level of involvement of a careful reader into preparation of a mathematical text and rendered the great service to the authors.

We would like to thank David Trarah for the efforts he made to clarify and strengthen the structure of the book and for his careful editing work which went much further than the usual copyediting. We also thank Sarah Shea-Simonds and Eugenia Kelbert for checking the style of presentation.

The book comprises three chapters divided into sections. Chapters 1 and 2 include material from Cambridge undergraduate courses but go far beyond in various aspects of Markov chain theory. In Chapter 3 we address selected topics from Statistics where the structure of a Markov chain clarifies problems and answers. Typically, these topics become straightforward for independent samples but are technically involved in a general set-up.

The bibliography includes a list of monographs illustrating the dynamics of development of the theory of random processes, particularly Markov chains, and parallel progress in Statistics. References to relevant papers are given in the body of the text. References to Vol. 1 mean *Probability and Statistics by Example*, Volume 1.