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ALAIN BENSOUSSAN

*University Paris Dauphine*



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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK

40 West 20th Street, New York NY 10011-4211, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

Ruiz de Alarcón 13, 28014 Madrid, Spain

Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

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First published 1992

First paperback edition 2004

*A catalogue record for this book is available from the British Library*

ISBN 0 521 35403 X hardback

ISBN 0 521 61197 0 paperback

## Contents

<i>Preface</i>	vii
1 Linear filtering theory	1
1.1 Filtering theory in discrete time	2
1.2 Filtering theory in continuous time	11
2 Optimal stochastic control for linear dynamic systems with quadratic payoff	19
2.1 A brief review of the deterministic systems	20
2.2 Optimal stochastic control with complete observation	23
2.3 Optimal stochastic control with partial information: simplified approach	29
2.4 Complete solution of the optimal stochastic control problem with partial information	36
3 Optimal control of linear stochastic systems with an exponential-of-integral performance index	53
3.1 The full observation case	54
3.2 The partial observation case	59
3.3 Additional remarks to the partial information case	72
4 Non linear filtering theory	74
4.1 Non linear filtering equation	76
4.2 Uniqueness theorem	89
4.3 Equation of the conditional probability	94
4.4 An explicit solution	101
4.5 Correlation between the signal noise and the observation noise	105
4.6 Some representation formulas for the conditional probability	114
4.7 Study of stochastic PDEs	126
4.8 Concluding remarks	135
5 Perturbation methods in non linear filtering	136
5.1 Linear systems with small noise in the observation	138
5.2 Non linear systems with small noise in the observation	146
5.3 Dynamic systems with small noise and small signal to noise ratio	167
5.4 Non linear filtering for dynamic systems with singular perturbations	177

6	Some explicit solutions of the Zakai equation	190
6.1	Non gaussian initial condition	192
6.2	Explicit solution in the case of a non linear drift	197
6.3	The conditionally gaussian case	204
7	Some explicit controls for systems with partial observation	222
7.1	The separation principle	223
7.2	The Bellman equation for the separated problem when $U_{ad}$ is bounded	227
7.3	Solution of the stochastic control problem with partial information when $U_{ad}$ is bounded	233
7.4	Solution of the Bellman equation in some particular cases, with bounded controls	238
7.5	Solution of the predicted-miss and minimum distance problems	246
7.6	An extension of the concept of solution	253
8	Stochastic maximum principle and dynamic programming for systems with partial observation	268
8.1	Setting of the problem	270
8.2	Stochastic maximum principle	276
8.3	Applications of the stochastic maximum principle	289
8.4	Preliminaries to dynamic programming	297
8.5	Stationary dynamic programming	305
8.6	Non stationary dynamic programming	314
8.7	Non linear semigroup	323
9	Existence results for stochastic control problems with partial information	326
9.1	Notation: setting of the problem	327
9.2	Stochastic optimal control	330
9.3	Existence of a solution	336
	<i>References</i>	340
	<i>Index</i>	351

## Preface

The problem of stochastic control of partially observable systems plays an important role in many applications. All real problems are in fact of this type, and deterministic control as well as stochastic control with full observation can only be approximations of the real world. This justifies the importance of having a theory as complete as possible, which can be used for numerical implementation.

In the first three chapters of this book we study problems which can be dealt with directly by algebraic manipulations, without using the complete theory. This is because the system and the observation have linear dynamics, and the cost is either quadratic or the exponential of a quadratic functional.

In Chapters 4 to 6, we present the theory of non linear filtering, which is the basic step in formulating the control problem adequately. The main difficulty, especially from the point of view of numerical applications, is that there are no statistics which are finite dimensional, and the basic object to be computed is the conditional probability. This is the solution of a stochastic partial differential equation (PDE) studied in Chapter 4. Chapters 5 and 6 are devoted to approximations, when perturbation methods are applicable, or to particular cases when simplification occurs, and sufficient statistics exist.

In Chapter 7, we study stochastic control problems with partial information, in an intermediate case, namely when the direct methods of Chapters 1, 2, 3 are not applicable yet the full theory is not necessary, either because finite dimensional sufficient statistics are available, or approximations are possible. In Chapter 8, we present the stochastic maximum principle and dynamic programming approach to the problem of stochastic control with partial information in the general case, which implies infinite dimensionality. In Chapter 9 we state, in a limited framework, some existence results.

Needless to say, I make no pretence to be exhaustive, and in this presentation have favoured analytic techniques, rather than algebraic and probabilistic ones. Reference to relevant literature is given throughout the book, for further study of these aspects. Numerical methods are not considered at all (see Legrand (1989) and Bensoussan and Runggaldier (1987) for more in this direction).