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CHAPTER 1

Overviews

1.1 Our objectives and approaches

This book is about modeling a large collection of interacting agents, and examining aggregate (deterministic) dynamics and associated stochastic fluctations.

There are two aspects or components in carrying out these objectives: dynamics, and random combinatorial analysis. The former is more or less self-explanatory and familiar to economists, although some of the techniques that are presented in this book may be new to them. The latter involves some facts or results that are rather new in economics, such as obtaining statistical distributions for cluster sizes of agents by types. More will be said on types later.

We regard economic processes as jump Markov processes, that is, continuous-time Markov processes with at most countable state spaces, and analyze formations of subgroups or clusters of agents by types. Jump Markov processes allow us to model group sentiments or pressure, such as fashion, fads, bandwagon effects, and the like. A cluster is formed by agents who use the same choices or decisions. Agents are thus identified with the rules they use at that point in time. Agents generally change their minds – that is, types – over time. This aspect is captured by specifying a set of transition rates in the jump Markov processes. Distributions of cluster sizes matter, because a few of the larger clusters, if and when formed, approximately determine the market excess demands for whatever goods are in the markets. There are some new approaches to firm size distributions as well.

Dynamics are represented by the master equations (the backward Chapman–Kolmogorov equations) for the joint probability distributions of suitably defined states of the collection of agents. The solutions of the master equations give us stationary or equilibrium behavior of the model and some fluctuations about them, obtained by solving the associated Fokker–Planck equations. Nonstationary solutions give us information on the time profiles of interactions, and how industries or sectors of economies mature or grow with time. These solutions may require some approximations, such as expansion of the master equations in some inverse powers of a parameter that represents the size of the model. In discussing multiple equilibria, we introduce a new equilibrium selection criterion, and consider distributions of the sizes of the associated basins of attractions in some random mapping contexts.

To fulfill our objectives, we use concepts such as partition vectors as state vectors, which arise in examining patterns formed by partitions of agents by types; Stirling numbers of the first and second kind, which have roots in combinatorial analysis; and distributions such as the Poisson–Dirichlet distributions and the multivariate Ewens distribution. All of these are unfamiliar to traditionally trained economists and graduate students of economics. We therefore present these as well as some others, as needs arise, to advance and support our modeling tasks and views proposed in this book.

Our approach is finitary. We start with a finite number of agents, each of whom is assumed to have a **choice set** – a set of at most countably many decision rules or behavioral rules. We define a demographic profile of agents composed of fractions of agents of the same type, with a finite number of total agents. We may let the number of agents go to infinity later, but we do not start with fractions of uncountably many agents arranged in a unit interval, for example (a typical starting point of some models in the economic literature). Our finitary approach is more work, but we reap a greater harvest of results. We may obtain more information on the natures of fluctuations, and more insight into dynamics, which get lost in the conventional approach.

Here is our approach in a nutshell. We start with a collection of a large, but finite, number of microeconomic agents in some economic context. We first specify a set of transition rates in some state space to model agent interactions stochastically. Agents may be households, firms, or countries, depending on the context of the models. Unlike examples in textbooks in probability, chemistry, or ecology, the reader will recognize that our transition rates are endogenously determined by considerations of value-function maximization associated with evaluations of alternative choices that confront agents.

Then we analyze the master equations that incorporate specified transition rates. Their size effects may be important in approximate analysis of the master equations. Stationary or nonstationary solutions of the master equations are then examined to draw their economic implications.

In models that focus on the decomposable random combinatorial aspects, distributions of a few of the largest order statistics of the cluster size distributions are examined to examine their economic consequences.

1.2 Partial list of applications

A number of models, mostly elementary or simple, are presented in Chapters 4 through 11 to illustrate the methods and potential usefulness of the proposed approaches. Some more elaborate models are found in parts of

Chapters 8 through 11, with some additional supporting material for them in Chapter 10.

One of the important consequences of our efforts is the derivation of aggregate, or emergent, or evolutionary patterns of behavior of large collections of agents. We deduce macroeconomic or sectoral properties or behavior of the microeconomic agents, starting from probabilistic descriptions of individual agents. Thus, we go part way from the microeconomic specifications of models to macroeconomic models, a level of models called **mesoscopic** by van Kampen (1992). By mesoscopic, we mean that we can deduce (nearly) deterministic average behavior and associated fluctuations. For example, we may think of building sectoral models composed of a large number of firms as mesoscopic models. Several or many such mesoscopic models may then be connected, or aggregated, as macroeconomic models.

We examine with fresh and different views processes for diffusion of new ideas or practices among firms in an industry, due to innovation, changing economic and social environments, disturbances, or the appearance and spread of new products among firms of an industrial sector, such as new manufacturing processes, new inventions, new employment policies, and technical improvements. We also examine a well-known model in the search literature, due to Diamond (1982), from our finitary perspectives in Chapter 9. In taking a new look at the Diamond model, we show a new probabilistic equilibrium selection criterion. In evaluating the Kiyotaki–Wright model (Kiyotaki and Wright 1993), which has similar dynamic structure to the Diamond model, we provide more dynamic analysis, and respecify their model so that money traders hold several units of money. Here, we show how partition vectors may be applied.

There are obviously many new results we can obtain in the area of industrial organization, such as entry and exit problems, changing market shares (Herfindahl index), or distributions of firm sizes or growth-rate distributions.

We also present, in Chapter 11, a simplified account of power laws that govern distributions of large price differences or returns in prices of some financial assets, and explanations of volatility switchings observed in financial time series, by examining conditions under which two large subgroups of agents with two opposing strategies form such that they largely determine the market excess demands for some financial assets.

1.3 States: Vectors of fractions of types and partition vectors

Levels of detail in describing behavior of a large collection of agents of several (or countably many) types dictate our choice of state variables.

Models are depicted in terms of **configurations**, namely patterns, or **states**, in more technical terms, and how they or some functions of them evolve with time. If we start our modeling task by specifying how sets of interacting agents behave at the level of microeconomics, then we may next inquire how some subsets or subgroups of microeconomic agents behave by attempting to describe their behavior in terms of less detailed state variables. These more **aggregated** state variables, or variables averaged over some larger subsets of agents than the original configurations, refer to model behavior at the aggregated, or more macroeconomic, levels of description. We use rather less detailed, or less probabilistic, specifications of states. Indeed, one of the insights we gain after many model-building exercises is that some of the very detailed microeconomic description found in some of the economic literature disappears, or matters less, as we describe agent behaviors averaged over larger sets of microeconomic configurations.

At the highest level of aggregation we have macroeconomic models in terms of macroeconomic variables. At a less aggregated level, we may have sectoral models described in terms of sectoral variables, which are less aggregated than the macroeconomic variables, but are more aggregated than microeconomic variables.[1] Stochastic description in terms of macroeconomic variables imply deterministic laws and the fluctuations about them (van Kampen 1992, p. 57).

We seek to link models for collections of microeconomic agents, whose behavior are described or stated by microeconomic specifications, with the aggregate or global behavior, which corresponds to mesoscopic or macroeconomic description.

1.3.1 Vectors of fractions

In this book, we use discrete states and models with finite or at most countable state spaces. This choice of state spaces is based on the way we describe microeconomic models and the details with which we describe behavior of agents – or, more pertinently, the decisions or choices they make, or the way we aggregate or incorporate microeconomic agents into macroeconomic models.

An example may help to clarify what we have in mind. At this preliminary stage of our explanation, let us suppose that agents have binary choices, or there are two types of agents, if we associate types with choices. The binary choices may be to participate or not in some joint projects, or to buy or not to buy some commodity or stocks at this point in time, etc. The nature of choices varies from model to model and from context to context. Here, we merely illustrate abstractly the ways states may be introduced. The two choices may be labeled or represented by 1 and 0, say. Then the state of n agents could be $\mathbf{s} = (s_1, s_2, \ldots, s_n)$, where $s_i = 1$ or $s_i = 0$, $i = 1, 2, \ldots, n$.

[1] To refer to variables at these intermediate levels we borrow a term from van Kampen (1992, p. 185) and call them **mesoscopic** variables for mesoscopic models. According to him, a mesoscopic quantity is a functional of the probability distribution (of states). He distinguishes mesoscopic variable from macro-variables.

This vector gives us a complete picture of who has chosen what. Thus, with regard to the information on the choice patterns by n agents, we don't need, nor can we have, more detail than that provided by this state vector. This is the microeconomic state at a point in time. We may then proceed to incorporate mechanisms or interaction patterns that determine how they may revise their choices over time, by specifying reward or cost structures and particulars on externalities among agents.

In some cases, we may decide not to model the collection of agents with that much detail. For example, identities of agents who have chosen 1 may not be relevant to our objectives of constructing models. We may care merely about the fraction of agents with choice 1, for example. Then, $\sum_i s_i/n$ is the information we need. Then we may proceed to specify how this **demographic** or fractional compositional information of agents evolves with time. At this level of completeness of describing the collection of a set of agents, the vector (n_1, n_2), where n_i is the number of agents with choice $i = 1, 2$, is a state vector. So is the vector made of fractions of each type of agents. This vector is related to the notion of empirical distribution in statistics. If the total number of agents is fixed, then the scalar variable n_1 or $f_1 = n_1/(n_1 + n_2)$ serves as the state variable.

With K choices or types, where K is larger than 2, detailed information on the choice pattern is provided by the vector $\mathbf{s}$, where s_i now takes on one of K possible values, and choice patterns may be represented by the vector of demographic fractions, or by a vector $\mathbf{n} = (n_1, n_2, \ldots, n_K)$, where n_j is the total number of agents making the jth choice.

1.3.2 Partition vectors

This choice of state vector may look natural. There is, however, another possibility. To understand this, let us borrow the language of the occupancy problem in probability, and think of K unmarked or indistinguishable boxes into which agents with the same choices (identical-looking balls) are placed. Let a_i be the number of boxes with i agents in them. With n agents distributed into K boxes, we have $\sum_{j=1}^{n} j a_j = n$, and $\sum_{j=1}^{n} a_j := K_n \leq K$. The first equation counts the number of agents, and the second the number of occupied boxes.

The vector with these as as components is a state vector for some purposes. In dealing with demographic distributions such as the number of firms in various size classes, the numbers of employees, the amount of sales per month, and so on, we are not interested in the identities of firms but in the number of firms each size class, as in the histogram representations of the numbers of firms of given characteristics or categories.

In some applications, we are faced with the problem of describing sets of partitions of agents of the type called exchangeable random partitions by

Kingman (1978a,b). The notion of partition vectors, in Zabell (1992), is just the right notion for discussing models in which some types of agents play a dominant role in determining market demands. This notion is discussed in detail in Chapter 3, and applied in Chapter 11 among many places.

We have briefly mentioned two alternative choices for state vectors. One of them is in terms of fractions of agents of each type or category. Instead of this more obvious choice of state variables, Watterson (1976) has proposed another way of describing states, which is less detailed than the one above using **n**. A level of disaggregation, or a way of describing the delabeled composition of a population of agents, is proposed that is suitable in circumstances in which new types of agents appear continually and there is no theoretical upper limit to the number of possible types. This is the so-called sampling-of-species problem in statistics (see Zabell 1993). The state of a population is described by the (unordered) set of type frequencies, i.e., fractions or proportions of different types, without stating which frequency belongs to which type. In the context of economic modeling, this way of description does not require model builders to know in advance how many or what types of agents are in the population. It is merely necessary to recognize that there are k distinct types in his sample of size n, and that there are a_j types with j agents or representatives in the sample. Compositions of samples and populations at this level are given by vectors **a** and **b** with components a_j and b_j, respectively, such that

$$\sum_{j=1}^{n} j a_j = n, \qquad \sum_{j=1}^{n} a_j = k,$$

and

$$\sum_{j=1}^{n} j b_j = n, \qquad \sum_{j=1}^{n} b_j = K$$

in the population, where N is the number of agents in the population, and K the number of distinct types, categories, or choices in the population, both being possibly infinite.

1.4 Jump Markov processes

By associating types with the decisions or choices, we may think of groups in which each agent has several alternative decisions to choose from. Agents may change their types when types are associated with their decisions, actions, or choices. In open models, agents of various types may, in addition, enter or leave the group or collection. These changes of fractions may occur at any time, not necessarily at the equally spaced discrete points of discrete dynamics. They are therefore modeled by continous-time (jump) Markov processes with finite or countable state spaces. See Norris (1997).

Among Markov processes we use those with a finite or at most countable states, and time running continuously. They are called jump or pure jump Markov processes in the probability literature.

1.5 The master equation

Once states have been assigned to a collection of economic agents, their behavior over time is specified by the dynamics for the joint probabilities of the states. Dynamics are set up by taking account of the probability fluxes into and out of a specified state over a small interval of time. We use the backward Chapman–Kolmogorov equation to do this accounting. We adopt the shorter name that is used in the physics and ecology literature and call the dynamic equation the **master equation**. This is an appropriate name because everything of importance we need to know about the dynamic behavior can be deduced from this equation. In particular we derive the dynamics for aggregate variables, which we call the **aggregate** dynamics (roughly corresponding to macroeconomic dynamics) and the dynamics for the fluctuations of state variables about the mean, or aggregate, values. The latter is called the Fokker–Planck equation.

It should be emphasized that the master equation describes the time evolution of the probability distribution of states, not the time evolution of the states themselves. This distinction may seem unimportant to the reader, but it is a crucial one and helps to avoid some technical difficulties. For example, in a model with two types of agents of a fixed total number, the fraction of one type of agents is often used as the state. The master equation describes how the probability for the fraction of one type evolves with time, not the time evolution of the fraction itself. The latter may exhibit some abnormal behavior at the extreme values of zero and one, but the probability distribution cannot.

When the master equations admit stationary solutions, as some models in this book do, we can deduce much from those distributions. Some nonstationary distributions may be obtained by the method of probability generating functions, or information on moments derived from cumulant generating functions. These are discussed in Chapters 3 and 4.

In general, we use Taylor series expansion in inverse powers of some measure of the model size, such as the number of agents in the model. We can show that in the limit of an infinite number of agents we recover traditional macroeconomic results. This is illustrated by reworking the well-known Diamond (1982) model in our framework in Chapter 9.

1.6 Decomposable random combinatorial structures

How do agents cluster or form subgroups in a market? What are the distributions of fractions on (high-dimensional) simplexes? These questions essentially have to do with random combinatorial structures such as random partitions. We borrow from Watterson (1976), Watterson and Guess (1977), and, more recently, Arratia (1992) and others to deal with the questions of multiplicities of microeconomic states compatible with a set of observations of (macroeconomic or mesoscopic) variables. In the second longest chapter of this book, Chapter 10, we connect three types of transition rates with three types of distributions, and discuss dynamics of clusters. Some of the results are then applied in Chapter 11, in which the two largest groups are on the opposite sides of the market and their excess demands drive the price dynamics of the shares.

1.7 Sizes and limit behavior of large fractions

We use order statistics of the fractions in some of the later chapters of this book. These have a well-defined limit distribution, called the Poisson–Dirichlet distribution, as the number of agents goes to infinity. The probability density of the first few of the fractions is later used in our discussion of approximations of market excess demands by a few dominant fractions in Chapters 10 and 11.

CHAPTER 2

Setting up dynamic models

This book is about setting up and analyzing economic models for large collections of interacting agents. We describe models in terms of states: as stationary distribution of states, or dynamics for time evolution of states. We may speak informally also of patterns of partitions of the set of agents by types or categories, or configurations, and how they or some functions of them evolve with time. Our models are usually specified as jump Markov processes, that is, Markov processes with a finite or at most countable number of states, and time running continuously.

In setting up a stochastic model for a collection of agents, then, we first choose a set of variables as a state vector for the model. The state vector should carry enough information about the model for the purpose at hand, so that we can, in principle, calculate the conditional distributions of future state vectors, given the current one.[1] Put differently, we must be able to calculate the conditional probability distributions of the model state vector at least for a small step forward in time, given current values of the state vector and time paths of exogenous variables.

This dynamic aspect of the model is described by the master equation, which is introduced in Chapter 3. Briefly, the master equation is the differential (or difference) equation that indicates how the probability of the model being at some specific state at a point in time is changed by the inflows and outflows of the probability fluxes. The name originates in the physics literature; see van Kampen (1992, p. 97).

The master equation is specified once the relevant set of transition rates for the model states is determined. Specifying these transition rates replaces the usual microeconomic specification of models.

[1] Strictly speaking, states should be (conditional) probability distributions, as pointed out by Bellman (1949). Informally and for convenience, however, we speak of a state vector even when we should speak of the distribution of the vector as the state.

In this chapter, we mention two basic types of state vectors we use in this book.

2.1 Two kinds of state vectors

Here, we begin by introducing two types of state vectors, called empirical distributions (frequencies) and partition vectors. Since the latter type is not used in the economic literature, we discuss it here and indicate why it is needed.

With K choices or types, where K is larger than 2, detailed information on the choices by individual agents is provided by the vector $\mathbf{s}$, where s_i now takes on one of K possible values, and choice patterns may be represented by the vector of demographic fractions, or by $(n_1, n_2, \dots, n_K)$, where n_j is the total number of agents making the jth choice.

As touched on in Section 1.2, there is, however, another possibility when types or choices do not have any inherent labels. The key is whether agents or categories are distinguishable or not. Do labels carry intrinsic information, or do they serve as mere labeling? The situation is exactly the same as that of the occupancy problem in which distinguishable or indistinguishable balls are to be placed in distinguishable or indistinguishable boxes. With identical-looking balls in boxes with no labels or distinguishing marks, how do we count the number of possible patterns of ball placements?

Suppose that K boxes, into which agents with the same choices are allocated, are indistinguishable. Let a_i be the number of boxes with i agents in them. With n agents distributed into K boxes, we have $\sum_{j=1}^{n} j a_j = n$ and $\sum_{j=1}^{n} a_j = K$. The first equation counts the number of agents, and the second the number of boxes. In Section 1.2 we introduced the partition vector. The partition vector $\mathbf{a}$ with these a's as components is a state vector for some purposes. It is called the partition vector by Zabell (1992) in the statistics literature, and is called the allelic vector by Kingman (1980) in the population-genetics literature. Sachkov (1996, p. 82) refers to it as the secondary specification of states. We give several examples in later chapters.

In other examples our interest in modeling lies not so much in the manner n agents are partitioned or clustered among different groups or types, which is captured by the frequencies, as in some structural properties of the patterns of partitions, for example, in the patterns of frequency variations. These are what are called frequencies of frequencies in the literature of ecology or population genetics.

We use probabilities that are invariant with respect to permutations of agents, because we regard agents as interchangeable. Empirical distributions are invariant under permutations of agents. Random partitions are equiprobable when their partition vectors are the same. These matters will be taken up in this