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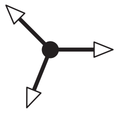
1 Moments

In this chapter you will learn
- what is meant by the moment of a force
- about the conditions for equilibrium of a rigid body

Key points from Mechanics 1

- A particle is an object that can be treated as if it were a point, so that all forces acting on it act at the same point.
- A set of forces acting at a point is said to be in equilibrium if the resultant is zero. If the forces are in equilibrium, the total component in any direction is zero.
- The normal reaction, of magnitude R, is the force at right angles to a surface which the surface exerts on an object in contact with it.
- A rough surface can offer a friction force parallel to the surface. This friction force has a maximum magnitude given by $F_{max} = \mu R$, where μ is the coefficient of friction.
- When the magnitude F of the friction force on an object is at its maximum, so that the object is about to move, the object is said to be in limiting equilibrium. If the object is in equilibrium but not limiting equilibrium, then $F < \mu R$.
- An object of mass m kg has a weight of mg newtons ($g = 9.8\,\text{m s}^{-2}$).

A Moment of a force (answers p 146)



In the situations studied in Mechanics 1, we looked at forces acting on a particle. In this case all the forces act at the same point.

If the object is not a particle, it is possible for the forces acting on it to be applied at different points.

For example, imagine that a ruler is placed on a smooth horizontal table and you are looking down on it.
In the first diagram below, two forces that are equal and opposite act on the ruler at the same point.
In the second diagram, the same two forces act at different points.

D **A1** Describe what happens to the ruler in each case.

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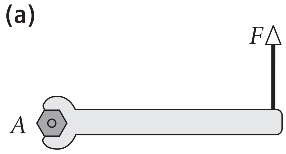
In the second case the two forces are equal and opposite but are not in equilibrium. Because their **lines of action** are different (in fact parallel), the effect on the ruler is different – it will turn.

The ruler is an example of a **rigid body**: this means an object that is not distorted by the forces acting on it – it stays the same shape and size.

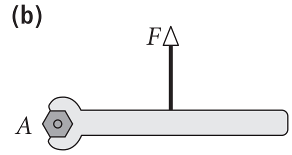
D

A2 A spanner is used to loosen a nut. A force is applied to the spanner as shown. Describe the effect of the force in each case shown.

(a)



(b)



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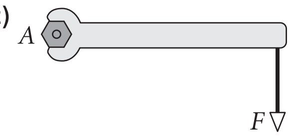


The turning effect of a force depends not only on the magnitude of the force but also on its point of application. If the magnitude stays the same but the line of action is moved further away from the point A about which the spanner turns, the turning effect is increased. The turning effect will be either clockwise or anticlockwise, depending on the direction of the force.

The turning effect about A is measured by the product of the magnitude of the force and the distance of the line of action from A.
This quantity is called the **moment** of the force about A.

Moment of force F about point A = magnitude of $F \times$ distance of line of action from A

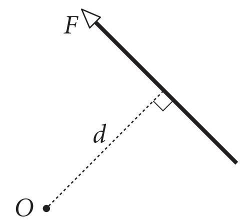
K

The moment of a force about a point is the product of the magnitude of the force and the perpendicular distance from the point to the line of action of the force.

Moment of F about $O = Fd$

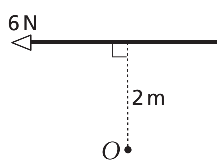
The **sense** of a moment is either clockwise or anticlockwise. Anticlockwise moments are taken as positive and clockwise moments as negative.

The units of a moment are newton metres (N m).

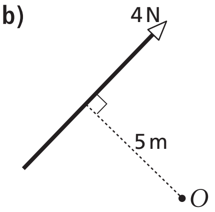


A3 Find the moment of each force about point O.

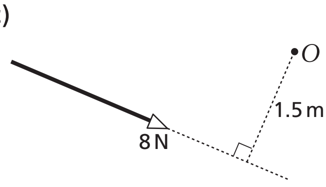
(a)



(b)



(c)

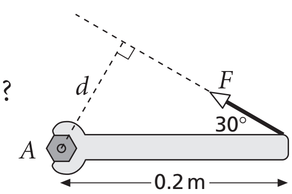


A4 A force F is applied as shown to the same spanner as in A2.

(a) Will the turning effect be more, less or the same as in A2 (a)?

(b) What is the perpendicular distance, d, from the point A to the line of action of F?

(c) Given that the magnitude of F is 8 N, find the moment of F about A.



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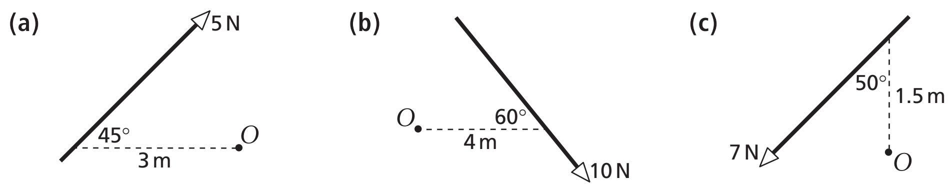
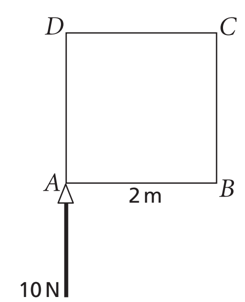
A5 Find the moment of each force about O.



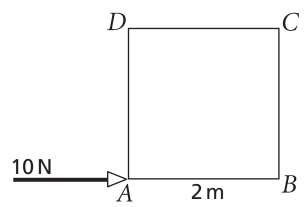
A **lamina** is an object which can be modelled as a plane area with mass but negligible thickness.



ABCD is a square lamina of side 2 m.
A force of 10 N acts at vertex *A* as shown.
If the lamina were fixed at vertex *B*, the force would cause a turning effect about *B*. This is the moment of the force about *B*.
The moment of the force about *B* is $10 \times 2 = 20$ N m clockwise, or -20 N m.

A6 Calculate the moment of the 10 N force about

(a) *C* **(b)** *D* **(c)** *A*



A7 The force is moved so that it acts as shown.
Calculate the moment of the 10 N force about

(a) *A* **(b)** *B*

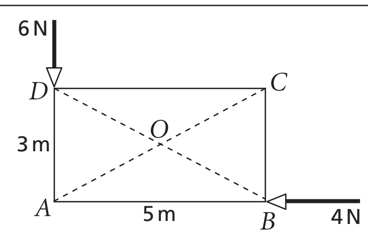
(c) *C* **(d)** *D*

K If the line of action of a force acts through a point, then the moment of the force about that point is zero.

Example 1

Forces of 4 N and 6 N act on the lamina *ABCD* as shown.

Find the total moment of these forces about *O*.



Solution

If O were fixed, the 4 N force would cause the lamina to move clockwise and the 6 N force would cause it to move anticlockwise.
The line of action of the 4 N force is 1.5 m from O and the line of action of the 6 N force is 2.5 m from O.

Moment of 4 N force about $O = 4 \times 1.5 = 6$ N m clockwise

Moment of 6 N force about $O = 6 \times 2.5 = 15$ N m anticlockwise

The total moment is the sum of the moments of the individual forces.
Remember that anticlockwise moments are positive and clockwise moments are negative.

Total moment $= 15 - 6 = 9$ N m

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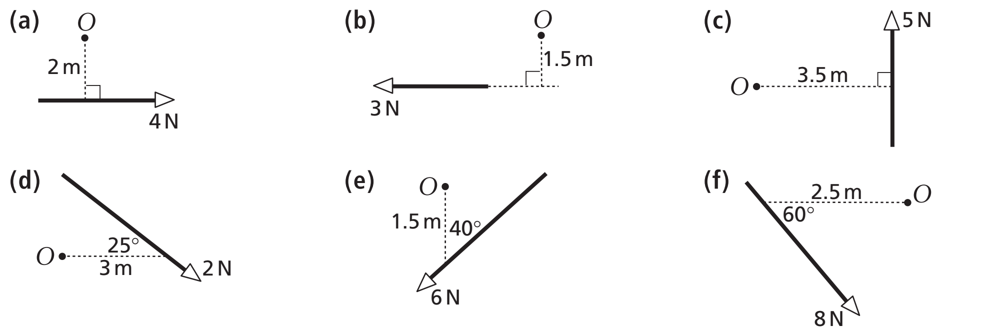
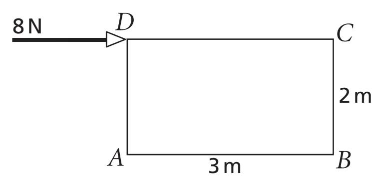
Exercise A (answers p 146)

1 Find the moment of each force about O.



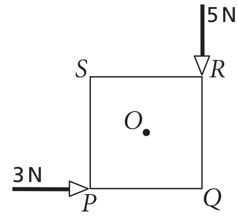
2 $ABCD$ is a rectangular lamina.
A force of 8 N acts at D as shown.
Find the moment of this force about



(a) A **(b)** B
(c) C **(d)** D

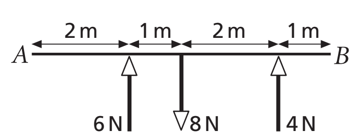
3 $PQRS$ is a square lamina of side 5 m with centre O.
Forces of 3 N and 5 N act as shown.
Find the total moment of these forces about

(a) P **(b)** S **(c)** O



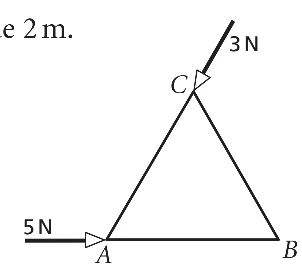
4 AB is a light rod of length 6 m.
Forces act on the rod as shown.
Find the total moment of these forces about

(a) A **(b)** B

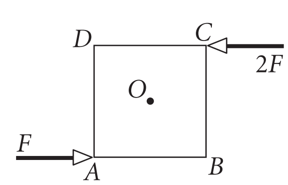


5 ABC is a light rigid equilateral triangular framework of side 2 m.
Forces act on the framework as shown.
Find the total moment of these forces about

(a) A **(b)** B **(c)** C



6 $ABCD$ is a square lamina of side 3 m.
Forces of magnitude F N and $2F$ N act on the lamina as shown.
The total moment of these forces about the centre O is 27 N m.
Find the value of F.



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B Equilibrium of a rigid body (answers p 146)

Kieran 20 kg

Sasha 25 kg

Imagine these two children playing on a seesaw of length 4 m and mass 30 kg, pivoted at its mid-point. They want to get the seesaw to balance.

D **B1** **(a)** Describe what happens to the seesaw if they sit at equal distances on either side of the central pivot.

(b) Who do you think should move to try to get the seesaw to balance?

The seesaw can be modelled as a **uniform rod**, that is a body which can be modelled as a straight line with its mass concentrated at its mid-point.
The point on a body at which its mass can be considered to be concentrated is called the **centre of mass** of the body.
The centre of mass of a uniform rod is at its mid-point.

The pivot, P, of the seesaw is at its mid-point.

Suppose that Kieran sits at one end of the seesaw, that is 2 m from P, and that Sasha's distance from P is x m.

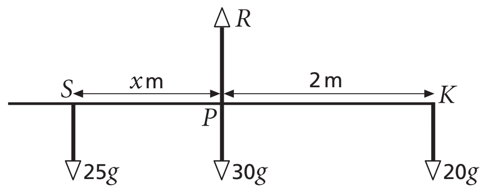
Taking the seesaw and children as one composite object, the forces in newtons acting on the object are

the weight of the seesaw, $30g$ acting at its centre

Kieran's weight, $20g$

Sasha's weight, $25g$

the upward reaction R of the pivot



The force diagram for the seesaw is shown.

We need to find the value of x for which the seesaw is in equilibrium.

For equilibrium, the total anticlockwise moment about P must equal the total clockwise moment about P.

The forces R N and $30g$ N go through P, so they have zero moment about P.

So $\quad 25g \times x = 20g \times 2$

$$\Rightarrow \quad x = \frac{40g}{25g} = 1.6$$

So Sasha must sit 1.6 m from the pivot.

Additionally, for equilibrium, the resultant of the forces on the seesaw must be zero.

B2 Use the fact that the resultant force on the seesaw is zero to find R.

When a particle is in equilibrium, the resultant of the forces acting on it is zero.
This condition is not sufficient for a rigid body to be in equilibrium.
A rigid body is in equilibrium if the resultant force is zero and the total moment is zero.

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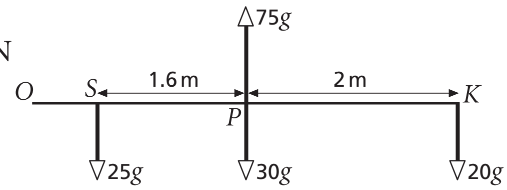
As the seesaw is in equilibrium, if the pivot is replaced by a force with the same magnitude as the reaction R N the seesaw will remain in equilibrium.

The force diagram for the seesaw is unchanged.



B3 **(a)** Find the total moment of the forces about S.

(b) Find the total moment of the forces about K.

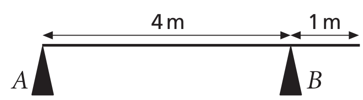
(c) Find the total moment of the forces about O.

(d) Comment on your results.

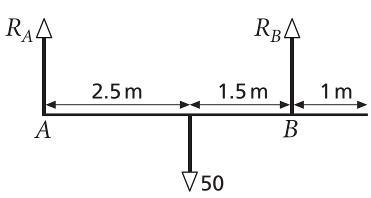
K A rigid body is in equilibrium if the resultant force is zero and the total moment about any point is zero.

Note that all the sets of forces in this chapter are coplanar, that is they act in the same plane.

B4 A uniform rod of weight 50 N rests horizontally in equilibrium on two smooth supports at A and B as shown.



The force diagram for the rod is shown. R_A and R_B are the magnitudes of the reactions at A and B.



The rod is in equilibrium, so the total moment about any point is zero.

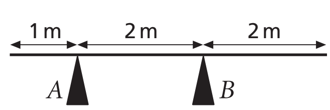
(a) Use the fact that the total moment of all the forces about A is zero to write down an equation involving R_B.
Hence find the value of R_B.

(b) Now do the same for the total moment about B, and hence find the value of R_A.

(c) Resolve the forces vertically to check that they are in equilibrium.

Finding the total moment about a point A is usually called 'taking moments about A'.
This can be abbreviated to M(A).
If the total moment about A is zero, it follows that the sum of the anticlockwise moments about A is equal to the sum of the clockwise moments about A.

B5 The supports are moved to the positions shown and the rod remains in equilibrium.



(a) Draw a force diagram for the rod.

(b) By taking moments about B, find the reaction at A.

(c) By taking moments about A, find the reaction at B.

(d) What effect has moving the supports had on the reactions?

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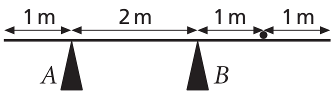
B6 A particle of weight 10 N is positioned on the rod, 1 m from B as shown. The rod remains in equilibrium.



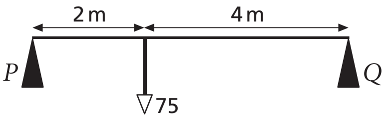
(a) Draw a force diagram for the rod.

(b) Find the reaction at A.

(c) Find the reaction at B.

A **non-uniform rod** is a body which can be modelled as a straight line with its mass concentrated at some point other than its mid-point.
The centre of mass of a non-uniform rod is generally not at its mid-point.

B7 A non-uniform rod PQ of length 6 m and weight 75 N is resting horizontally on supports at P and Q.
The centre of mass of the rod is 2 m from P.



(a) Draw a force diagram for the rod.

(b) Find the reaction at P.

(c) Find the reaction at Q.

B8 A particle of weight 30 N is positioned at the centre of rod PQ.

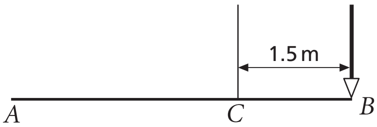
(a) Find the reaction at P.

(b) Find the reaction at Q.

K A problem about a rigid body in equilibrium can be solved by recognising that the total moment about an appropriate point equals zero and that the forces resolved in any direction equal zero.
Taking moments about a point through which an unknown force acts can eliminate the need to find that force.

Example 2

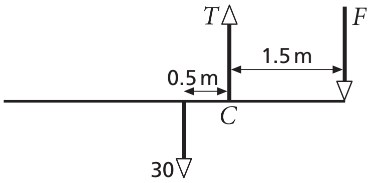
A uniform rod has weight 30 N and length 4 m.
It is suspended by a light string attached at point C.
The rod is held in equilibrium by a vertical force applied at the end B as shown.



Find the magnitude of the applied force.

Solution

Sketch a force diagram showing all the forces in newtons acting on the rod.



The question does not require the tension to be found, so take moments about C to set up an equation without T.

M(C): $30 \times 0.5 = F \times 1.5$
$\Rightarrow \quad F = 10$

The magnitude of the applied force is 10 N.

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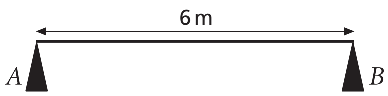
Example 3

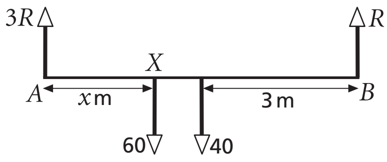
A uniform rod of weight 40 N rests horizontally in equilibrium on two smooth supports at A and B as shown. A particle of weight 60 N is placed on the rod at X. The reaction on the rod at A is three times the reaction at B. Find the distance AX.



Solution

Sketch a force diagram showing all the forces acting on the rod. The reaction at A is three times the reaction at B, so label these forces 3R and R.

The distance AX, in metres, is labelled x so XB = 6 – x.



Take moments about A. M(A): $60x + 40 \times 3 = 6R$

$\Rightarrow$ $60x + 120 = 6R$ (1)

Take moments about B. M(B): $3R \times 6 = 60(6 - x) + 40 \times 3$

$\Rightarrow$ $18R = 480 - 60x$ (2)

Multiply (1) by 3. $18R = 180x + 360$ (3)

Subtract (2) from (3). $0 = 240x - 120$

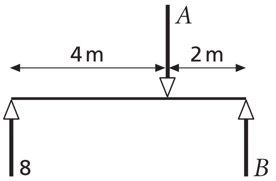
$x = 0.5$

The distance AX is 0.5 m.

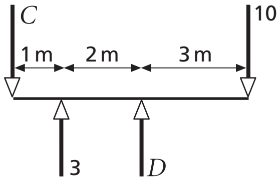
Exercise B (answers p 147)

1 Each diagram below shows a light rod in equilibrium under the action of coplanar parallel forces in newtons. Find the values of the lettered forces.

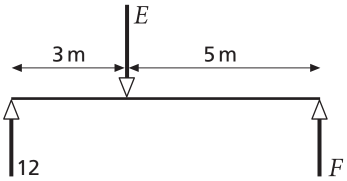
(a)



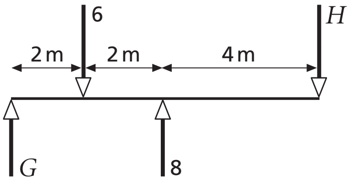
(b)



(c)

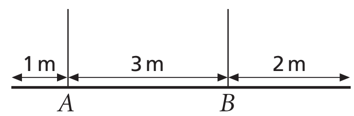


(d)



2 A seesaw of length 6 m is pivoted at its mid-point *P*.
Khyle sits on one side of the seesaw, 3 m from the pivot, and Jack sits on the other side, 2.4 m from the pivot. The seesaw is in equilibrium.
Given that Khyle has mass 32 kg, find the mass of Jack.

3 A uniform rod of weight 12 N and length 6 m hangs in equilibrium in a horizontal position.
It is held in position by two light vertical cables attached at *A* and *B* as shown.

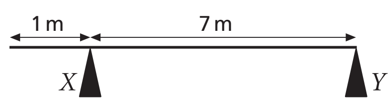


(a) Draw a force diagram for the rod.

(b) Find the tension in the cable at *A*.

(c) Find the tension in the cable at *B*.

4 A uniform rod of weight 14 N rests in equilibrium on supports at *X* and *Y* as shown.



(a) Find the reaction at *X*.

(b) Find the reaction at *Y*.

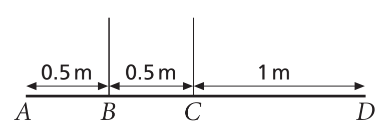
5 A non-uniform rod *AB* of length 5 m and weight 100 N rests horizontally in equilibrium on supports at *X* and *Y* as shown.
The centre of mass is 2 m from *B*.

(a) Draw a force diagram for the rod.

(b) Find the reaction at *X*.

(c) Find the reaction at *Y*.

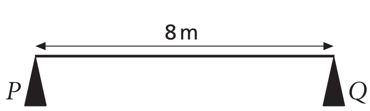
6 A non-uniform rod *AD* of length 2 m and weight 25 N hangs in equilibrium in a horizontal position.
It is held in position by two light vertical cables attached at *B* and *C* as shown.
The centre of mass of the rod is 0.6 m from *A*.



(a) Find the tension in the cable at *B*.

(b) Find the tension in the cable at *C*.

7 A non-uniform rod of weight 60 N rests horizontally in equilibrium on supports at *P* and *Q* as shown.
The reaction at *P* is four times the reaction at *Q*.
Find the position of the centre of mass of the rod.



8 A uniform plank *AC* of mass 55 kg and length 10 m rests horizontally in equilibrium on supports at *B* and *C* as shown.

A child of mass 20 kg stands on the plank at *A*. The plank remains in equilibrium.
The reaction on the plank at *B* is now twice the reaction at *C*.
Find the distance *AB*.

C Tilting (answers p 147)

D

C1 A uniform beam rests horizontally in equilibrium on two smooth supports as shown. Tim stands on the beam.
Describe what might happen to the beam if Tim stands between

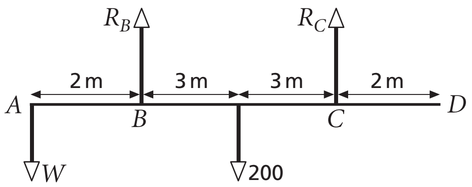
(a) *A* and *B* **(b)** *B* and *C* **(c)** *C* and *D*

The beam may or may not remain in equilibrium depending on several factors including Tim's weight, the weight of the beam, the length of the beam and where the supports are placed.

The beam *AD* is of length 10 m and weight 200 N. The supports at *B* and *C* are positioned symmetrically 2 m from *A* and *D* respectively.
Tim stands on the beam at *A*.

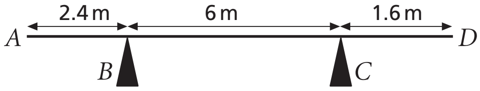
The force diagram for the beam is shown.

The reactions at the supports *B* and *C* will vary depending on Tim's weight, *W* N.



C2 **(a)** By taking moments about *B*, find an equation that links *W* and R_C.
(b) Describe what happens to R_C as *W* increases.
(c) What is the maximum value of *W* for the beam to remain in equilibrium?
(d) Describe what happens to R_B as *W* increases.
(e) Write down the values of R_B and R_C as the beam is about to tilt.

C3 The supports for the beam are moved and it now rests horizontally in equilibrium as shown. Neeta stands on the beam at *D* and it just remains in equilibrium.



(a) Which of the supports will be the pivot if the beam tilts?
(b) By taking moments about this pivot, find Neeta's weight.
(c) Find the magnitudes of the reactions at *B* and *C*.

K

When a rod or beam is about to tilt about one support, the reaction of the other support on the rod is zero.

When a suspended rod is about to tilt about one support, the tension at the other support is zero.