

## Symbol index

Except when otherwise noted, all Lie groups  $G, K, \dots$  are connected and simply connected:  $\mathfrak{g}, \mathfrak{k}, \dots$  are the corresponding Lie algebras.

Common symbols (not defined in text):

$A \sim B$  is the set-theoretic difference.

$K \backslash G$  is the set of cosets  $Kg$ ;  $G/K$ , the set of cosets  $gK$ .

$\mathfrak{g}/\mathfrak{k} = \mathfrak{k} \backslash \mathfrak{g}$  is the set of additive cosets  $\mathfrak{x} + \mathfrak{k}$ .

$A \approx B$  means that  $A, B$  are homeomorphic or diffeomorphic spaces.

$\mathfrak{m}^\perp, \mathfrak{k}^\perp, \dots$  are the annihilators in the dual space of  $\mathfrak{m}, \mathfrak{k}, \dots$

$\mathcal{C}_c, \mathcal{C}_c^\infty$  are used for spaces of continuous ( $C^\infty$ ) functions with compact support.

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