

## Index

- 
- absorbent neighbourhood, 34
  - active constraint, 13
  - adjoint bifunction, 24
  - adjoint equation, 161
  - adjoint perturbation function, 24
  - adjoint variable, 161
  - affine hull, 38
  - aligned, 74
  - alternative hypothesis, 9
  - arc of network, 173
  - augmented Lagrangian, 147, 195
  
  - backlogging, 163
  - balanced neighbourhood, 33
  - Baire category theorem, 179
  - barrelled space, 192
  - biadjoint bifunction, 85
  - biadjoint perturbation function, 85
  - biconjugate function, 87
  - bidual bifunction, 24, 85
  - bidual perturbation function, 85
  - bifunction, 16, 17, 190
  - bifunctional duality, 44, 81
  - binding constraint, 13
  - Brouwer's fixed point theorem, 136
  - $\beta$ -topology, 36
  
  - calculus of variations, 162
  - canonical mapping, 180
  - capacitated network, 174
  - circled neighbourhood, 33
  - Chebyshev approximation, 3, 154
  - closed function, 90
  - closed loop control, 162, 195
  - closure of function, 90
  - coarse(r) topology, 34
  - compatible spaces, 26, 37
  - complementarity (problem), 107, 108, 192
  - complementary slackness, 13, 70
  - complete space, 187
  - concave closure, 96
  - concave conjugate, 89
  - concave function, 32, 33
  - cone, 30, 193
  - cone generated at point of set, 49, 127
  - cone of internal directions, 125
  - cone of tangents, 115, 124
  - conjugate function, 87, 192
  - constraint, 2
  - constraint qualification, 61, 193
  - constraint set, 2
  - continuous functional, 17, 176
  - contraction, 133
  - control, 5
  - converse duality, 84, 96, 192
  - convex bifunctional programming, 44
  - convex combination, 30
  - convex function, 32, 33
  - convex hull, 30
  - convex process, 193
  - convex programming, 28
  - convex set, 30
  - countable additivity, 63
  
  - decision space, 1, 17
  - decision variable, 1
  - degeneration of constraints, 97
  - differential game theory, 189
  - distinguishing points, 34
  - dual bifunction, 24
  - duality, 22
  - duality gap, 43
  - dual negative cone, 32
  - dual of dual, 85
  - dual pair of spaces, 34
  - dual perturbation function, 24
  - dual positive cone, 32
  - dual problem, 23
  - dual solution, 49
  - dual space, 12
  - Dubovitskii–Milyutin 'formalism', 191
  - dynamic programming, 9, 28, 168, 173, 191, 195
  
  - effective domain, 33

*Index*

203

- epigraph, 33  
 equality constraint, 2, 63, 64, 114, 126, 129, 190  
 equilibrium point, 132, 194  
 equivalence of dualities, 81  
 error of the first kind, 9  
 error of the second kind, 9  
 expected cost, 8  
 extended Fenchel duality, 79  
 extension of mapping, 177
- Farkas' lemma, 114, 171  
 feasible, 1  
 feasible region, 2  
 feasible solution, 1  
 feedback, 162  
 Fenchel duality, 72, 82  
 Fenchel duality extended, 79  
 Fenchel transform, 87  
 fine(r) topology, 34  
 finite intersection property, 142  
 first-order approximation of sets, 125  
 first-order conditions, 113  
 fixed point of mapping, 132, 194  
 fixed point theorems, 133, 136, 137, 138, 194  
 fixed time optimal control, 6, 156  
 Fréchet derivative, 111  
 Fréchet differential, 111  
 Fréchet differentiable, 111
- game, 7  
 game theory, 132, 189  
 Gâteaux differentiable, 101  
 generalized complementarity, 108  
 geometric programming, 190  
 Glicksberg, 138  
 global conditions, 12, 15, 26  
 global properties, 110, 118  
 Gordon's theorem of the alternative, 172  
 graph of function, 137
- Hamiltonian, 159  
 Hamiltonian equations, 161  
 Hausdorff distance, 133  
 hypograph, 96
- indicator function, 71  
 induced constraint, 63  
 inequalities, 4, 153, 195  
 inequality constraint, 2, 59, 114, 127, 129, 190
- infeasible (problem), 49  
 infimum, 1  
 infinite linear programming, 10  
 integer programming, 10, 28  
 internal point, 178  
 inventory control, 8, 163  
 inverse function theorem, 193  
 isolated local minimum, 148
- Kakutani's fixed point theorem, 137, 138  
 Karush–Kuhn–Tucker conditions, 190  
 Kuhn–Tucker conditions, 27, 109, 190  
 Ky Fan, 138
- Lagrangian (function), 8, 19, 56, 102  
 Lagrangian duality, 59, 82  
 level sets, 140  
 linear continuous functional, 17  
 linearization lemma, 112, 184, 193  
 linearized set, 110, 124  
 linear programming, 5, 10, 24, 28, 61, 69, 195  
 linear variety, 38  
 locally convex topological vector space, 33  
 local optimality conditions, 27  
 local point of view, 27  
 local properties, 110, 118  
 lower semi-continuity, 88, 137  
 l.s.c., 137  
 l.s.c.–u.s.c., 142
- Mackey topology, 36  
 mathematical programming, 9, 27  
 minimax theorem, 141, 194  
 minimum, 1  
 minimum norm problem, 3, 74  
 minimum principle, 27, 110, 159, 193  
 Minkowski-functional, 178  
 mixed constraint, 66  
 modified Lagrangian, 147, 195  
 Motzkin's theorem of the alternative, 169  
 multifunction, 133  
 multi-objective function, 1, 173  
 multi-objective optimization, 10, 173  
 multiplier, 12, 28  
 multiplier space, 17
- negative cone, 31  
 network, 173  
 node of network, 173  
 nonlinear programming, 28  
 nonnegativity constraints, 5

- normal bifunction, 43  
 normality, 43  
 normal problem, 43  
 null-hypothesis, 9  
 nullspace, 184
- objective function, 1  
 one-sided directional derivative, 98  
 open loop control, 162, 195  
 open mapping theorem, 65, 180  
 operations research, 188, 195  
 optimal control, 5, 156, 189, 190, 194  
 optimal solution, 1, 49  
 optimization problem, 1  
 oriented network, 174
- Pareto optimality, 173  
 partial dualization, 66  
 perturbation, 11, 28, 190  
 perturbation function, 12, 17  
 perturbation space, 17  
 point-to-set mapping, 132, 137  
 polar of set, 32, 35  
 polar topology, 36  
 Pontryagin minimum principle, 27, 159  
 positive cone, 30  
 positively homogeneous, 98  
 power set, 132  
 primal problem, 23  
 primal solution, 49  
 production planning, 5, 189  
 pseudo convex, 191  
 purely finitely additive, 62
- quadratic programming, 106  
 quasi-concave, 140  
 quasi-convex, 140, 191  
 quasi-convex-concave, 140  
 quotient space, 180
- R*-closed function, 95  
 recourse, 8  
 reflexive space, 121, 127–9  
 regularity condition, 61, 114  
 relative interior, 38, 179  
 retracting mapping, 138  
*R*-normal bifunction, 95
- saddle-point (of Lagrangian), 8, 20, 56, 104, 132  
 saddle-value (of Lagrangian) 56  
 second-order conditions, 119, 128, 194
- seminorm, 178  
 separating hyperplane, 37  
 separation theorems, 38, 39, 191  
 simplex method, 172  
 sink of network, 173  
 slack variable, 70  
 Slater's constraint qualification, 61  
 Slater's theorem of the alternative, 172  
 solution, 1  
 solvable (problem), 1, 49  
 source of network, 173  
 space of decisions, 1  
 space of perturbations, 16  
 space with positive cone, 30  
 Sperner's lemma, 135  
 $\sigma$ -topology, 34  
 stably set, 52  
 state (of optimal control), 5, 157  
 state variable of dynamic programming, 29  
 statistical decision theory, 189  
 Stiemke's theorem of the alternative, 172  
 stochastic optimization problem, 8, 163, 189  
 strict complementarity, 150  
 strict local minimum, 121, 148  
 strictly convex, 134, 140  
 strong(er) topology, 34  
 strong separation, 37  
 subdifferentiability, 18, 99, 190  
 subdifferential, 99  
 subgradient, 18, 99  
 support function, 87, 178  
 supporting hyperplane, 42, 87  
 supremum, 3  
 $\sigma(Z, Z^*)$ -topology, 34  
 $\sigma(Z^*, Z)$ -topology, 35
- test function, 9  
 theorem of the alternative, 169, 195  
 theory of games, 7  
 topological vector space, 17  
 transposition theorem, 169  
 transversality conditions, 13, 162  
 Tucker's theorem the of alternative, 171  
 Tychonoff, 138
- unconstrained problem, 2  
 upper semi-continuity, 63, 65, 137  
 u.s.c., 63, 65, 137

*Index*

205

variational inequality, 105, 192  
vertical hyperplane, 43, 96  
weak(er) topology, 34, 35  
weak separation, 37

weak\* topology, 35  
Young transform, 87  
zero-sum game, 132