

Index

- Absolutely continuous spectrum 50, 86, 97
- Automorphism 6
 - Kolmogorov, K. 50–52, 54, 62, 67–69
 - Bernoulli 57, 58, 62, 63
 - Markov 53–55, 58, 62, 63
- Average, phase 2
 - time 2, 12
- Axioms, of information theory 33
- Bernoulli, shifts 53
 - automorphisms 53, 58, 62, 63
 - endomorphisms 53
 - theory 58, 59
- Birkhoff's
 - ergodic theorem 2, 3, 25
- Boltzmann 2
- Canonical decomposition 96
- Classification, theory 56
 - problem 57, 58, 62
 - theorem of Halmos and von Neumann 56–57
- Cocycle additive 74, 75
- Computations of entropy 41, 66, 71
- Completely positive entropy 64, 66, 68, 69
- Conditional, expectation 20
 - probability 21
 - information 33
 - entropy 34
- Conservative mechanical system 1
- Continuous, eigenfunction 82–84
 - spectrum 46, 48, 78, 97
- Covariance, singular 72
- Cross-section, global 6
- Cyclic subspace (cycle) 92
 - maximal 95
- Decomposition, of operators 46, 95–97
 - of Hilbert spaces 95–97
 - of functionals 13
 - lemma 23
 - canonical 96
- Decreasing Martingale theorem 30
- Deterministic completely 60
- Direct product of partitions 62
 - spaces 32, 41, 46, 53
 - transformations 32, 41, 46, 53, 61, 70
- Discrete spectrum 45, 46, 57, 82, 96, 97
- Distal 72
- Ehrenfest 2
- Eigenfrequency 78
- Eigenfunction 45, 48, 78, 96
- Eigenvalue 45, 96
- Endomorphism 6
 - exact 52
 - Bernoulli 53
 - Markov 53
- Entropy 34, 56
 - conditional 34
 - completely positive 64, 66, 68, 69
 - computations 41, 63, 71
 - zero 71
- Ergodic hypotheses 2
 - measure (uniquely) 14
- Ergodicity 43–47
- Ergodic theorems
 - Birkhoff's 23, 25, generalisations 4, of information theory 39
 - L^p 27, maximal 24
 - scope of 3
 - topological 12, 14, 15
 - von Neumann's 2, 3, 21
 - Wiener's dominated 25
- Exact endomorphisms 52
- Examples 74
- Existence of invariant measures 4, 14
- Extensions Z_2 85
- Extreme points 14
- Factor 56
- Fourier
 - series 18, 47, 79, 82
- Generator 59
 - strong 59

Index

109

- Gibbs 1, 3, 19
- Global cross-section 6
- Group invariants 57, 76
- Hamiltonian 1, 2
- Herglotz's theorem 45, 90
- Hilbert space 21, 43, 90
- Homomorphism 56
- Identically distributed 27
- Increasing Martingale theorem 30
- Independence of functions 27
 - of partitions 33, 36
- Information, axioms 33
 - conditional 33
 - ergodic theorem 39
 - theory 33
- Isometry 20, 45
- Isomorphism 56
 - invariants 57
 - weak 56
- Invariant
 - function 1, 17, 21, 23
 - isomorphism 57, 59
 - Kolmogorov–Sinai 60
 - measure 2, 7, 14
 - set 14, 21
 - spectral 45, 47, 50, 96
- Invertible 6
- Jensen's inequality 35
- Khintchine's
 - recurrence theorem 22
- Koopman 2
- Kronecker 19
- Kolmogorov 4, 5, 58
 - K-automorphism 50–52, 54, 62, 67–69
- Kolmogorov–Sinai invariant 60
- L^p ergodic theorem 28
- Law of large numbers (strong) 27, 32, 33
- Lebesgue (semi-Lebesgue) spectrum 50, 51, 52, 62, 85
- Liouville 1
- Markov chains 52
 - automorphisms 53–55, 58, 62, 63
 - endomorphisms 53
 - shifts 53
- Martingales
 - decreasing 30
 - increasing 30
- Maximal, cycle 92
 - ergodic theorem 24
 - lemma 29
 - spectral type 95
- Mean ergodic theorem 28
- Measure
 - ergodic 14
 - (existence of) invariant 2, 7, 14
 - preserving 6
 - spectral 45, 93
- Mechanical systems 1, 2
- Minimal 16, 72, 84
- Mixed spectrum 46
- Mixing
 - all orders 52
 - strong 49
 - topologically weak 82
 - weak 47
- Multiplication operator 93
- Multiplicity function (spectral) 96
- Null set (conventions) 7, 8
- Operators
 - decomposition of 46, 95–97
 - isometric 20, 45
 - multiplication 93
 - unitary 20, 45, 90
- Ornstein 5, 58
- Orbit 16
- Origins 1
- Partitions 33
- Phase average 2
- Pinsker σ -algebra 64
- Poincaré map 7
 - recurrence 3, 19
- Point (or discrete) spectrum 45, 46, 57, 82, 96, 97
- Polynomials (uniform distribution of) 11, 17, 18
- Positive definite 90
- Probability 6
- Quasi-discrete spectrum 57
 - ergodic hypothesis 2
- Relatively dense 22
- Recurrence, Gibbs 19
 - Poincaré, 19
 - Khintchine 22
- Riesz representation theorem 13
- Rohlin–Sinai theorem 68
- Shannon–McMillan–Breiman theorem 39
- Shifts 52
- σ -algebra
 - Pinsker 64
 - trivial 21, 50, 64
- Singular covariance 72
 - spectrum 71, 72
- Spectral equivalence 45, 56, 93

110

Index

- invariant 45, 47, 50, 96
- measure 45, 93
- multiplicity function 96
- theory 90
- type 95
- Spectrum, absolutely continuous 50, 86, 97
 - continuous 46, 48, 78, 97
 - discrete 45, 46, 57, 82, 96, 97
 - Lebesgue 50, 51, 52, 62, 85
 - point 45, 46, 57, 82, 96, 97
 - singular 71, 72
- Statistical mechanics 1
- Strong-generator 59
- Strong law of large numbers 27, 32, 33
- Strong-mixing 49
- Strongly separating 72
- Structural stability 5
- Time average 2, 12
- Topological dynamics 9
 - ergodic theorem 12
 - weak-mixing 82
- Trivial σ -algebra 21, 50, 64
- Uniform distribution mod 1 9
- Uniquely ergodic 15, 17, 84
- Unitary operator 20, 45, 90
- Unitarily equivalent 45, 56, 93
- Velocity change 74, 75, 80–82
- von Neumann's
 - ergodic theorem 2, 3, 21
- Weak isomorphism 56
 - mixing 47, 49, 87
- Weyl's criterion 9
 - theorem on polynomials 11, 17, 18
- Wiener's, dominated ergodic theorem 25
 - theorem 92
- Winding numbers 75–78
- Z_2 extensions 85