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IDEAL THEORY

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PREFACE

The theory of ideals, in its modern form, is a contemporary development of mathematical knowledge to which mathematicians of to-day may justly point with pride. It not only has the generality and purity of logical structure, which is typical of so much of the work that has been done in recent years, but also it has contributed, in a substantial way, to the growth of an older branch of the mathematical tree, namely, algebraic geometry. It is not possible, in a volume of this size, both to give a useful account of the purely algebraic parts of our subject and also to give examples of the deeper applications, but it has proved possible to weave into a connected algebraic theory those results which play outstandingly important roles in the geometric applications. This is precisely what has been done. It is the author's hope that this tract will extend the interest taken in a new mathematical territory, by enabling the reader to travel, in relative comfort, along the road which pioneers like E. Noether, W. Krull, C. Chevalley and I. S. Cohen have constructed. Before he sets out, however, either to see the sights or with the intention of joining in the work when he reaches his destination, the traveller may fairly ask whether or not his present equipment will be sufficient for the proposed journey. Provided he has reached the standard of a good honours degree in mathematics he has no cause to worry. It is not necessary that he should have ever read a book or attended lectures on modern algebra, for in this respect the account is self-contained.

The notes, which follow Chapter V, will give the reader a general idea of the historical development of our subject, but his picture of the way in which the theory has grown will be distorted unless he remembers that the topics discussed form only a part of what might have been included under the broad heading 'Ideal Theory'. For this reason, he will find no mention of the work of certain mathematicians whose results would inevitably occupy prominent and important positions in a comprehensive treatise. There is another respect in which the notes would be misleading without some further comment. They indicate, in detail, the sources from which the materials for the tract have been collected. What they do not reveal, however, is the great personal debt the author owes to Professor E. Artin of Princeton University, who, during the years 1946–8, introduced him to the theory of ideals and developed his interest in it. More recently, the well-known writings of O. Zariski on algebraic geometry have been responsible for arousing, in the author, a particular interest in the properties of local rings, and he ventures to hope that this little volume will assist the mathematician, who has not had an intensive training in modern algebra, in reading Professor Zariski's papers.

It is a great pleasure to acknowledge the help which was given by Professor W. V. D. Hodge, who read the whole manuscript. Both the general plan of this tract and a number of points of detail have been very much improved as a result of his suggestions. The decision to devote a substantial amount of space to developments made during the last decade was largely due to his influence.

I am also very grateful to Dr Christine M. Hamill for the assistance which she has given in correcting the proofs.

D. G. N.

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NOTE TO THE READER

The numbering of theorems, propositions and lemmas is begun afresh in each chapter. When a reference is made to a result, which has previously been established, only the number is quoted if the result in question is in the same chapter as that in which the reference is made. In all other cases, both the number of the result and the number of subsection, in which it occurs, are given.