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**ASYMPTOTIC EXPANSIONS**

# ASYMPTOTIC EXPANSIONS

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BY

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## CONTENTS

<i>Preface</i>	<i>page</i> vii
1. Introduction	1
2. Preliminaries	5
3. Integration by parts	13
4. The method of stationary phase	27
5. The method of Laplace	36
6. Watson's lemma	48
7. The method of steepest descents	63
8. The saddle-point method	91
9. Airy's integral	99
10. Uniform asymptotic expansions	107
<i>Bibliography</i>	118
<i>Index</i>	120

## PREFACE

In 1943, at the request of the Admiralty Computing Service, I wrote a short monograph on *The Asymptotic Expansion of a Function Defined by a Definite Integral or Contour Integral*. This was one of a series of monographs intended for use in Admiralty Research Establishments, on topics which appeared to be inadequately covered in easily accessible literature. It evidently met a need of the time, since a revised edition was issued in 1946 and had a wide circulation.

The Admiralty monograph has long been unobtainable, and several of my friends have urged me to write this more extensive book on the same general lines. There are few theorems; the aim is the modest one of explaining the methods which are available, and illustrating them by means of a few of the more important special functions.

I must express my thanks to Professor Arthur Erdélyi for the generous advice and encouragement he has given me during the writing of this book.

E. T. C.