

CONTENTS

<i>Art.</i>	<i>Page</i>
<i>Author's Preface</i>	v
 <i>Chapter I. SETS OF POINTS</i> 	
1·1 The algebra of sets	1
1·2 Infinite sets	3
1·3 Sets of points. Descriptive properties	4
1·4 Covering theorems	6
1·5 Plane sets	7
 <i>Chapter II. MEASURE</i> 	
2·1 Measure	10
2·2 Measure of open sets	10
2·3 Measure of closed sets	11
2·4 Open and closed sets	12
2·5 Outer and inner measure. Measurable sets	13
2·6 The additive property of measure	14
2·7 Non-measurable sets	15
2·8 Further properties of measure	16
2·9 Sequences of sets	18
2·10 Plane measure	21
2·11 Measurability in the sense of Borel	23
2·12 Measurable functions	23
 <i>Chapter III. THE LEBESGUE INTEGRAL</i> 	
3·1 The Lebesgue integral	26
3·2 The Riemann integral	27
3·3 The scope of Lebesgue's definition	28
3·4 The integral as the limit of approximative sums	30
3·5 The integral of an unbounded function	31
3·6 The integral over an infinite range	33

viii	CONTENTS	
<i>Art.</i>		<i>Page</i>
3·7	Simple properties of the integral	34
3·8	Sets of measure zero	37
3·9	Sequences of integrals of positive functions	38
3·10	Sequences of integrals (integration term by term)	40
 <i>Chapter IV. DIFFERENTIATION AND INTEGRATION</i> 		
4·1	Differentiation and integration as inverse processes	44
4·2	The derivatives of a function	44
4·3	Vitali's covering theorem	46
4·4	Differentiability of a monotonic function	48
4·5	The integral of the derivative of an increasing function	49
4·6	Functions of bounded variation	50
4·7	Differentiation of the indefinite integral	52
4·8	Absolutely continuous functions	54
 <i>Chapter V. FURTHER PROPERTIES OF THE INTEGRAL</i> 		
5·1	Integration by parts	58
5·2	Change of variable	58
5·3	Multiple integrals	61
5·4	Fubini's theorem	63
5·5	Differentiation of multiple integrals	65
5·6	The class L^p	65
5·7	The metric space L^p	67
 <i>Chapter VI. THE LEBESGUE-STIELTJES INTEGRAL</i> 		
6·1	Integration with respect to a function	70
6·2	The variation of an increasing function	71
6·3	The Lebesgue-Stieltjes integral	72
6·4	Integration by parts	75
6·5	Change of variable. Second mean-value theorem	77
	Solutions of some examples	80