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Steven E. Ingebritsen, Ward E. Sanford and Christopher E. Neuzil
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Chapter 1
Groundwater flow

In this chapter we introduce fundamental hydrogeologic concepts and derive the basic equations of groundwater motion. It seems appropriate to begin with *Darcy's law*, because we will generally assume that some form of this empirical law adequately describes rates of fluid flow. Groundwater flow rates calculated by Darcy's law are a unifying theme for this book in that they are used to describe fluid flow through a deforming volume, solute transport by advection and mechanical dispersion, and heat transport by advection. After introducing Darcy's law we will explore associated parameters such as *fluid potential*, *hydraulic head*, *porosity*, *hydraulic conductivity*, and *permeability*. We will then introduce the concept of the *water table* and *confined* and *unconfined* groundwater systems, briefly discuss the *elemental volume* or macroscopic approach that is implicit in the definition of hydrogeologic parameters, and proceed to derive the groundwater flow equation by considering conservation of mass and the production of fluid from "storage" in a geologic medium (introducing *specific storage* and *specific yield*). Finally, we will consider the assumptions embodied in various forms of the groundwater flow equation. It will become clear that the commonly invoked forms of the equation, posed in terms of hydraulic head, are too simplified or incomplete to apply to many geologic problems. Nonetheless, they need to be examined and understood as a common point of reference and departure. In succeeding chapters we will develop the coupled theory of groundwater flow and deformation (Chapter 2), solute transport (Chapter 3), and heat transport (Chapter 4), and then describe applications of flow and transport theory to geologic processes (Chapters 5 to 13).

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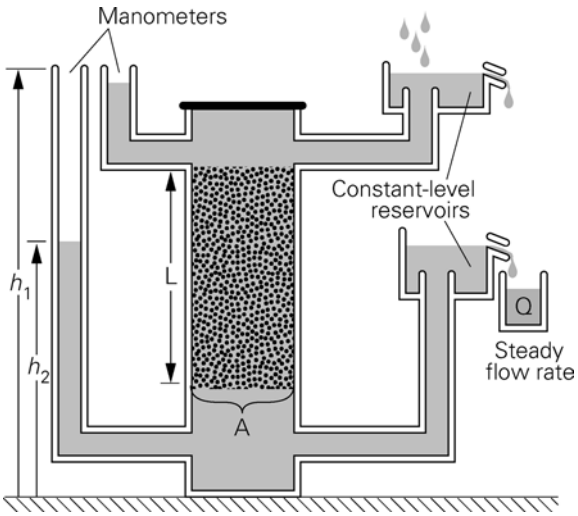

Figure 1.1. Schematic diagram of the apparatus used in Henry Darcy's sand-filter experiments.

1.1 Darcy's law

In 1856 the French engineer Henry Darcy published the results of a set of experiments aimed at developing design parameters for sand water filters. The experimental apparatus, shown schematically as Figure 1.1, allowed him to vary the length (L) and cross-sectional area (A) of a sand-packed column and also the elevations of constant-level water reservoirs connected to the upper (h_1) and lower (h_2) boundaries of the column. Under steady flow conditions, the volumetric flow rate (V_w/t) through the column (Q) was positively correlated with A and ($h_1 - h_2$) and inversely correlated with L. By introducing a constant of proportionality K, Darcy's experimental results can be summarized as

$$Q = \frac{K A(h_1 - h_2)}{L}. \tag{1.1}$$

We arrive at what has become known as *Darcy's law* by dividing both sides by A, writing $(h_1 - h_2)/L$ in differential form by defining $(h_1 - h_2) \rightarrow \mathrm{d}h$ and $L = (L_1 - L_2) \rightarrow \mathrm{d}L$, and introducing a minus sign to indicate that flow is in the direction of decreasing h, so that

$$q = -K\left(\frac{\mathrm{d}h}{\mathrm{d}L}\right), \tag{1.2}$$

where q denotes the volumetric flow rate per unit area and has units of L/t ($V_w/\mathrm{t}/\mathrm{L}^2 = \mathrm{L}^3/\mathrm{t}/\mathrm{L}^2 = \mathrm{L}/\mathrm{t}$). The constant of proportionality K, named *hydraulic conductivity*, must also have dimensions of L/t, because the hydraulic gradient $\mathrm{d}h/\mathrm{d}L$ is dimensionless.

Both theoretical considerations (Hubbert, 1940, 1956a) and laboratory experiments with a variety of fluids later revealed that K expresses a combination of fluid and solid properties. The flow rate is actually

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proportional to the specific weight of the fluid, $\rho_f g$, inversely proportional to the dynamic viscosity of the fluid, μ_f, and proportional to a property of the solid medium, k, which is called *intrinsic permeability*. Thus

$$K = \frac{k\rho_f g}{\mu_f}, \tag{1.3}$$

where k has units of L^2. Experiments with sand or glass beads of uniform diameter d_m further revealed that, for granular porous media, q, K, and k are proportional to d_m^2.

In hydrogeologic practice, the volumetric flow rate per unit area, q, is often termed the *specific discharge* or *Darcy velocity*. In this book we will sometimes use the term *specific discharge* but will also use the term *volume flux* to refer to q. The latter term is more physically descriptive, and a quantity so-named is logically multiplied by a fluid density or solute concentration to arrive at a mass flux.

Despite the velocity-like units, q is not an actual fluid velocity, because the fluid is flowing only through void (pore) spaces, not through the total cross-sectional area A. An estimate of the actual fluid velocity is obtained by dividing q by that fraction of the total area or volume that is composed of connected void space, that is,

$$v = \frac{q}{\left(\frac{V_v}{V_t}\right)} = \frac{q}{n_e}, \tag{1.4}$$

where v is called the *average linear velocity* or *seepage velocity*, V_v is the volume of connected void space, V_t is the total volume, and the ratio V_v/V_t defines the *effective porosity*, n_e. The average linear velocity is an estimate of the mean of the pore-scale fluid velocity. It represents, for example, the mean velocity at which a conservative (nonreacting) solute would move through Darcy's experimental column (Figure 1.1). In Darcy's experiments, $n_e = 0.38$ (Darcy, 1856), so $v = 2.6q$.

Darcy's law in the form of Eq. (1.1) or (1.2) is a simple *linear transport law*, completely analogous to the expressions that describe conduction of heat and electricity or the diffusion of a solute. For example, an analogous form of Fourier's law for conductive heat flow is

$$Q_h = -K_m A \left(\frac{dT}{dL}\right), \tag{1.5}$$

where Q_h is heat flow, K_m is thermal conductivity, and dT/dL denotes the temperature gradient. Similarly, Ohm's law for conduction of electrical

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current can be written

$$i = -K_e A \left(\frac{dV}{dL} \right), \tag{1.6}$$

where i is electrical current, K_e is electrical conductivity (the reciprocal of resistivity), and V is (temporarily) redefined here as voltage. Fick's first law of diffusion (Eq. 3.1) has the same form, relating the diffusive flux of a solute to its concentration gradient through a constant of proportionality.

Combining these linear transport laws with *continuity equations* that express mass or energy conservation results in *diffusion equations* that describe the transient flow of groundwater, heat via conduction, electricity, and solute via diffusion. In Section 1.5 we use this approach to derive the diffusion equation for groundwater flow. The full-fledged diffusion equations for electricity and conductive heat flow are not particularly relevant to the purposes of this chapter. However, those readers with some background in electrical or heat-flow theory should recognize and benefit from the analogy. Furthermore, one should be aware that many analytical solutions to diffusion-type equations are available in the heat-flow literature, as are many solutions to *advection–diffusion equations* similar to those that describe the coupled processes of groundwater flow and solute or heat transport (e.g., Carslaw and Jaeger, 1959; see Chapters 3 and 4 for solute and heat transport, respectively). With an appropriate substitution of parameters, the solutions in the heat-flow literature can be directly applied to groundwater flow and transport problems.

We regard Darcy's law as fundamentally empirical, although it may be arrived at by averaging more fundamental equations of fluid motion known as the *Navier–Stokes equations* over a representative volume of porous medium, assuming laminar, steady flow or negligible inertial terms, and making a few other assumptions (e.g., Hubbert, 1956a).

1.1.1 The limits of Darcy's law

Darcy's law has been tested empirically over a fairly wide range of conditions. Experimental results indicate that it fails at sufficiently high volume fluxes. Above certain threshold flow rates, the relationship between q and dh/dL becomes nonlinear and significant amounts of energy are lost to turbulence. As a result, Darcy's law overpredicts the flow rate associated with a given value of dh/dL. Such flow rates are exceedingly rare in the subsurface but can occur very near to a wellbore or in areas of cavernous porosity, such as occur in carbonate rocks and lava flows.

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The upper limit for application of Darcy's law is usually estimated on the basis of the dimensionless *Reynolds' number*. The Reynolds' number parameterizes the relative strengths of inertial and viscous forces and was first defined in a landmark paper on the transition from smooth, laminar flow to turbulent flow in cylindrical pipes (Reynolds, 1883). It can be written $Re = \rho_f q L/\mu_f$, where ρ_f is fluid density, q is the volume flux, μ_f is the dynamic viscosity of the fluid, and L is some characteristic length. In granular porous media, L is commonly related to the grain-size distribution (although it is also sometimes taken as $k^{1/2}$ (Ward, 1964), where k is the intrinsic permeability), and the transition to non-Darcian (nonlinear) flow appears to take place at $Re \sim 5$ (Bear, 1979; Freeze and Cherry, 1979). In fractured media the transition to non-Darcian flow can be estimated in the context of a parallel-plate model of fracture porosity ($k \sim Nb^3/12$, where N is the fracture spacing; Snow, 1968). In this approach, b would be taken as the fracture aperture, q is replaced by v, the average linear velocity through the fractures, and the critical Re value is about 1000 (Vennard and Street, 1975). Applying the porous-medium approach, evaluating ρ_w (1000 kg/m^3) and μ_w (1.1×10^{-3} Pa-s) for liquid water at 15 °C, and taking d_m as the median grain size (d_{50}) of a coarse sand (0.001 m), we find that Darcy's law would begin to overpredict flow rates for $q > \sim 6 \times 10^{-3}$ m/s or $\sim 2 \times 10^5$ m/yr. *(Note: Throughout this book, when we are discussing a particular fluid we use subscripts such as w, s, o to refer to specific fluids such as liquid water, steam, or oil. We use the subscript f in reference to generic fluids or fluid mixtures.)*

Although Darcy's law fails at high flow rates, a low flow rate or low permeability limit has not been demonstrated. Apparent departures from a linear relation between a hydraulic gradient dh/dL and the volume flux q (Eq. 1.2) were reported in the literature as early as 1899, mainly from experiments under low-gradient (e.g., King, 1899) and/or low-permeability conditions. Many of these experiments have since been shown to have been flawed or incorrectly interpreted (e.g., Olsen, 1965; Neuzil, 1986). However, nearly all experiments with intrinsic permeabilities less than about 10^{-15} m^2 have imposed very large values of dh/dL (Neuzil, 1986), rather than values representative of *in situ* conditions (dh/dL generally ≤ 1), because experiments with *in situ* gradients are impractically slow. Thus it is still reasonable to speculate that Darcy's law might not strictly apply in extremely low permeability environments. In most geologic media, low permeability is associated with extremely small pore and fracture openings, such that a large fraction of the pore water occurs in close proximity to hydrophilic solid surfaces. There are significant variations in some water properties with distance from solid surfaces (e.g., Mitchell, 1993); for example, there is a more rigid

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short-range order or "structure" among the water molecules nearer to a solid. If the narrowest connected openings are dominated by such "structured" water, there might be a finite threshold value of dh/dL below which Darcy's law does not apply (that is, $q = 0$ for some finite range of $dh/dL > 0$, or q does not vary linearly with dh/dL).

1.1.2 Driving forces for groundwater flow

In hydrogeologic practice the driving force for groundwater flow is generally expressed in terms of a parameter called *hydraulic head* or simply *head*. This is the same quantity indicated by h_1 and h_2 in the schematic of Henry Darcy's laboratory apparatus (Figure 1.1). The nature of hydraulic head was clarified by Hubbert (1940), who derived it from basic physical principles after stating with some asperity that "to adopt [head] empirically without further investigation would be like reading the length of the mercury column of a thermometer without knowing that temperature was the physical quantity being indicated."

Intuitively, one might tend to think of groundwater as flowing from areas of high pressure to areas of low pressure. This idea is readily debunked by consideration of the pressure distribution in a static column of water ($P = \rho g d$, where d is the depth below the water surface) or, alternatively, by simple experiments such as the one diagramed in Figure 1.2a, where flow is from $P = 0$ at point A to $P > 0$ at point B to $P = 0$ at point C. As Hubbert (1940) demonstrated, groundwater flow is actually from areas of high energy to areas of low energy.

Two fundamental forms of energy are of interest in this context: *kinetic energy* (KE) and *potential energy* (PE). Kinetic energy is associated with motion, and the kinetic energy of a unit volume of liquid water is given by $\rho_w v^2/2g$. Many readers will recognize that this quantity is very similar to the *velocity head* term of *Bernoulli's equation*. In a typical groundwater environment the kinetic energy is negligibly small relative to the total potential energy (KE is typically at least a factor of 10^{10} less than PE) and can be neglected.

Potential energy is associated with the work required to move from one place to another in a conservative force field, that is, a field in which the work done in moving from point A to point B does not depend on the path taken. In a groundwater context the most important conservative force fields are gravity and pressure. The *gravitational potential energy* of a unit volume of liquid water is $\rho_w g z$, where z is its height above an arbitrary datum and $\rho_w g$ is its *specific weight*. The *pressure potential energy* per unit volume is simply the pressure P, a force per unit area. For a fluid with variable density (ρ_f not a constant), flow is proportional to the gradient in the quantity $(P + \rho_f g z)$ (see Sections 3.3 and 4.1.3).

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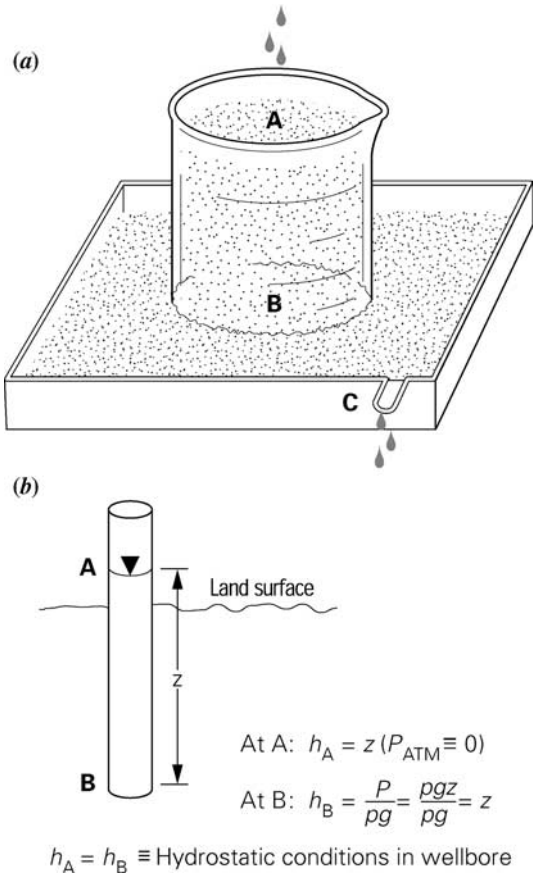

Figure 1.2. (a) A simple experiment demonstrating that groundwater does not necessarily flow from areas of high pressure to areas of low pressure. Water is added at a steady rate to an open-ended sand-filled column partly embedded in a sand-filled tray. Flow is from point A (P = atmospheric = 0) to point B ($P > 0$) to point C ($P = 0$). (b) Components of hydraulic head ($h = P/\rho_f g + z$) illustrated with reference to a *piezometer*, a tube that is open to the atmosphere at the top and to groundwater flow at the bottom. The pressure datum ($P = 0$) is taken as atmospheric pressure, and the bottom of the tube is the reference elevation datum where $z = 0$. At point A, then, $h_A = 0 + z = z$. The pressure at point B is determined by the weight of the overlying column of water, $\rho_w gz$, so that $h_B = \rho_w gz/\rho_w g + 0 = z$. The relation $h_A = h_B = z$ defines *hydrostatic* (nonflowing) conditions within the piezometer tube.

For an *incompressible fluid* (ρ_f constant), or one that is nearly so, one can divide through by the specific weight to obtain the quantity $(P/\rho_f g + z)$, which defines the *hydraulic head* h. Both the *pressure head* $(P/\rho_f g)$ and the *elevation head* (z) have units of length, and the total hydraulic head h can be equated to the water level observed in a manometer (Figure 1.1)

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Table 1.1 *Direct and coupled flow phenomena*

Direct flow phenomena appear on the main diagonal and coupled flow phenomena appear in the off-diagonal positions[a].

	Gradient			
Flow	Hydraulic head	Chemical concentration	Temperature	Electrical
Fluid	Darcian flow (*Darcy's law*)	chemical osmosis	thermoosmosis	electroosmosis
Solute	ultrafiltration	diffusion (*Fick's first law*)	thermal diffusion (*Soret effect*)	electrophoresis
Heat	isothermal heat transfer	Dufour effect	heat conduction (*Fourier's law*)	Peltier effect
Charge	streaming current (Rouss effect)	diffusion and membrane potential	thermoelectricity (*Seebeck effect*)	electrical conduction (*Ohm's law*)

[a] After Mitchell (1993) and Bader and Kooi (2005).

or well (Figure 1.2b). (See also Problem 1.1.) In this case the driving force for groundwater flow is the head gradient.

Linear flow laws such as Darcy's law describe what are termed *direct flow phenomena* (Mitchell, 1993), in which the driving force and flow are of like types (head gradients and volume flux, concentration gradients and solute flux, temperature gradients and heat flux). Often, simultaneous flows of different types occur when only one driving force is acting, for example fluxes of solutes and heat via fluid volume flux driven by gradients in hydraulic head. However, it is important to note that these are distinct from *coupled flow phenomena* in which gradients of one type (e.g., concentration gradients) cause flows of another type (e.g., fluid flow). Some direct and coupled flow phenomena are summarized in Table 1.1, where direct flow phenomena appear on the main diagonal and coupled flow phenomena appear in the off-diagonal positions.

Quantitative description of coupled flow phenomena typically is problematic. There is limited information about the values of off-diagonal coupling coefficients and it is difficult to design experiments to characterize them. Nevertheless, certain couplings have been demonstrated to be significant. *Streaming currents* (Table 1.1) are of interest because the rapid flow of electrolyte-bearing groundwater generates electrical anomalies that can reveal patterns of subsurface flow (e.g. Revil and others, 1997). *Electroosmosis* (Table 1.1) has been used to dewater fine-grained materials in engineering projects. The coupled flow of greatest interest in geologic systems, however, is *chemical osmosis*.

Chemical osmosis is water flow driven by yet another kind of potential energy, in this case differences in *chemical potential* (e.g. Robinson

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and Stokes, 2002) resulting from differences in solute concentration. It occurs in materials that behave as *semi-permeable membranes*, or media that impede the movement of solutes relative to water (see Section 3.6). Clays and clayey geologic materials often have such properties and evidence suggests that highly compacted claystones and shales can be very efficient membranes. Chemical osmosis occurs when the difference in chemical potential between high and low concentration solutions drives water toward the high concentration. The concentration difference also tends to drive solute toward the low concentration by diffusion, but the movement of solute is inhibited by the membrane. The resulting disparity causes a net movement of water toward the region of high concentration until the pressure there increases sufficiently to balance the difference in chemical potential. The resulting pressure excess is called *osmotic pressure* or, more properly, *osmotically induced pressure*. Strictly speaking, osmotic pressure is a property of a solution and equals the pressure difference that would be achieved with a *perfect membrane* that allows no solute movement at all, but the term has come to be used for any pressure change due to osmosis.

Chemical osmosis is described using an extended version of Darcy's law, namely

$$q = -K\left(\frac{\mathrm{d}h}{\mathrm{d}L}\right) + \sigma K\left(\frac{\mathrm{d}\pi_h}{\mathrm{d}L}\right) \tag{1.7}$$

(e.g. Bresler, 1973), where σ is the *membrane efficiency* or *reflection coefficient* and π_h is osmotic head, or the solution's osmotic pressure divided by $\rho_\mathrm{w} g$. Membrane efficiency takes values between 0 for no membrane effects and 1 for a perfect membrane.

The role and significance of chemical osmosis in natural groundwater systems has been debated for decades without a clear consensus arising. Membrane properties in clays and shales have been demonstrated by numerous laboratory studies (e.g. Kemper, 1961; Young and Low, 1965; Keijzer, 2000; Cey and others, 2001). However, few examples of chemical osmosis have been found in the subsurface, and most are ambiguous. Perhaps the most convincing was discovered in the Dunbarton basin of South Carolina and Georgia, where hydraulic heads approximately 130 m above hydrostatic are correlated with pore-fluid salinities of 12 000 to 19 000 mg/liter (Marine and Fritz, 1981). More recently, a nine-year *in situ* experiment in the Cretaceous Pierre Shale (South Dakota, USA) suggested the shale was capable of generating osmotic-pressure anomalies as large as 20 MPa which could persist for tens of millions of years (Neuzil, 2000). Chemical osmosis is of interest if it can achieve such pressures and, for example, affect crustal deformation (Chapters 2 and 9). Osmotic pressures could also masquerade as pressures maintained by active geologic forcing (Section 5.2.1), erroneously indicating ongoing

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crustal processes where there are none. However, because of the uncertainty attached to it, in this book we will generally neglect chemical osmotic driving forces and assume that groundwater flow is driven solely by gradients in hydraulic head or, more generally, by gradients in the quantity $(P + \rho_f g z)$.

1.2 Crustal permeability

Permeability is unquestionably the most crucial hydrologic parameter. Unfortunately, it is often a very difficult parameter to evaluate and apply in a meaningful fashion, especially over the enormous space and time scales that apply in many geologic problems.

The measured permeability of common geologic media varies by an almost inconceivable 16 orders of magnitude, from values as low as 10^{-23} m^2 in intact crystalline rock, intact shales, fault gouges, and halite to values as high as 10^{-7} m^2 in well-sorted gravels. Furthermore, permeability estimates often appear to vary with the scale or method of measurement, particularly in media where fracture permeability is significant. For example, in fractured rocks the relatively large-scale permeabilities measured by *in situ* wellbore testing are commonly several orders of magnitude higher than the smaller-scale permeabilities measured on the associated drill core. Finally, in many geologic problems permeability must be regarded as a time-dependent parameter, being created or destroyed over time by mineral dissolution and precipitation (Section 11.2), by changes in *effective stress* (Sections 2.1.2 and 9.1) that result in consolidation or *hydraulic fracturing* (Section 5.2.3), and by thermoelastic effects.

1.2.1 Permeability versus porosity

The terms porosity (n) and permeability (k) are often used interchangeably by nonhydrogeologists. This confusion is understandable, because there is a strong positive correlation between the two quantities in many porous and fractured geologic media. For well-sorted, unconsolidated, granular porous media, this correlation is often expressed by the *Kozeny–Carman equation* (Carman, 1956),

$$k = \frac{n^3}{\left[5 s_0^2 (1-n)^2\right]}, \tag{1.8}$$

where s_0 is the solid surface exposed to the fluid per unit volume of solid material. The Kozeny–Carman relation was developed by solving the *Navier–Stokes equations* for a system of parallel capillary tubes; in this model, s_0 can readily be related to the hydraulic radii of the

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