

## INDEX

- abelianization, 206
- Adams condition, 166
- Adams summand, 148
- Adams-Novikov spectral sequence, 182
- $A(\Gamma, k)$ , 194
- $A_\infty$  ring spectra, 3, 58, 135, 152
- algebraic  $K$ -theory of rings, 36
- André-Quillen cohomology
  - for differential graded  $E_\infty$   $k$ -algebras, 127
  - for simplicial  $E_\infty$   $k$ -algebras, 127
  - of  $C$ -algebras, 177
  - of algebras in  $E_*E$ -comodules, 178, 179
  - of unstable algebras over the Dyer-Lashof algebra, 191
- André-Quillen homology, 116
- AQ, 116
- associated  $K$ -theory spectra, 19
- associative ring spectra, 3
- Atiyah-Bott-Shapiro orientation
  - equivariant version, 88, 111
- Atiyah-Hirzebruch spectral sequence, 122, 124
- augmentation ideal, 117, 204
  
- Beilinson's Lemma, 47
- bipermutative categories, 19, 24
- Bott elements for  $KK$ , 104
- Bott periodicity in  $KK$ -theory, 103
  
- center of rings, 46
- cofibrant chain complex, 40
- commutative  $S$ -algebra, 13, 19, 201
- commutative ring spectra, 3, 24
- completed Johnson-Wilsen spectra, 148
- complex cobordism, 3, 71
- connective spectra, 19
- coordinate-free prespectra, 92
- coordinate-free spectra, 8
- cotangent complex
  - algebraic, 116
- cross-effect, 143
- cyclic homology, 46
  
- deformations of formal group laws, 194
- derivations
  - bimodule, 208
  - commutative, 208
- derived categories, 34, 39
  - of dg rings, 50
  - of rings, 34
- derived mapping space, 157
- $Der_{MU}^*(H\mathbb{F}_p, H\mathbb{F}_p)$ , 221
- $Der_S^*(H\mathbb{F}_p, H\mathbb{F}_p)$ , 220
- differential graded rings, 50
- distinguished triangles, 43
- Dwyer-Kan classification space, 160, 228
  
- $E(\Gamma, k)$ , 196
- Eilenberg-Mac Lane spectra, 69
- $E_\infty$  operad, 189
- $E_\infty$  ring spectra, 3, 10, 11, 13, 19, 58, 87, 152
- $E_\infty$  operad, 146
- endomorphism ring spectrum, 71
- equivalence
  - of the derived categories of two rings, 49
  - of triangulated categories, 46
- equivariant  $K$ -theory, 5
- equivariant  $Spin^c$ -cobordism, 88, 108
- equivariant orthogonal spectra, 88
- equivariant Swan theorem, 101
- étaleness, 119
- exact functor between triangulated categories, 46
- Ext-groups, 42
  
- free unstable algebras over the Dyer-Lashof algebra, 190
- $FSP$ 
  - equivariant, 88
- function spectra, 12, 66, 157, 203
  
- Gamma cohomology, 141
- Gamma homology, 115, 119, 126
  - chain complex for, 120, 144
- Gamma-spaces, 3
- generalized tilting theorem, 81
- generators
  - in a triangulated category, 44
  - in an abelian category, 34

234

*Index*

- of a stable model category, 59
- geometric realization
  - and algebra structures, 164
  - and spiral exact sequences, 171
- good objects, 38
- Grothendieck groups
  - of rings, 46
  - of triangulated categories, 46
- heart, 50
- $H\Gamma$ , 119
- higher  $K$ -groups of rings, 46
- Hochschild cochain complex, 136
- Hochschild homology, 46, 116
- Hochschild-Kostant-Rosenberg theorem
  - for  $S$ -algebras, 119
  - for algebras, 119
- homomorphism complexes, 52
- homotopy category of chain complexes
  - over a ring, 60
- homotopy groups
  - natural, 169
- $\mathcal{J}$ -FSP, 88, 91
- infinitesimal extensions, 117
- $K$ -projective complex, 41
- Kähler differentials, 116
- Kasparov  $K$ -theory
  - equivariant, 100, 101
- $K(M, n)$ , 177
- $K$ -theory spectra, 19
- lax morphisms, 21
- Lie representation, 141
- $\text{Lie}_n$ , 141
- linear isometries operad, 10
- Loday functor, 144
- $\mathbb{L}$ -spectra, 11
- Lubin-Tate spectra, 4, 148, 151, 194, 198
- map of type  $B_A(M, n)$ , 185
- mapping spaces
  - homotopy type, 180, 182, 226
- model category, 34
  - $E_2$ , 167
  - $\mathcal{P}$ -resolution, 168
  - on simplicial  $T$ -algebras, 173
  - of  $C$ -algebras, 158
- of operads in simplicial sets, 158
- of simplicial  $C$ -algebras, 177
- resolution, 167
- stable, 59
  - with a single small generator, 71
- module categories of rings, 33
- module categories over ring spectra, 34
- module of differentials
  - relative, 212
- module of indecomposables
  - algebraic, 116
  - for  $S$ -algebras, 117, 205
- moduli space
  - of  $C$ -algebra structures, 161
  - of  $n$ -stages, 183
  - of realizations, 153, 182, 228
- Morava  $E$ -theories, 151, 196
- Morava stabilizer group, 195
- Morava- $K$ -theory
  - $A_\infty$ -structures, 140
- Morita equivalence, 5, 35
- Morita theory, 33
  - for derived categories, 49
  - for ring spectra, 74
  - in abelian categories, 34
  - in stable model categories, 58
- $M \times A$ , 176
- $MU$ -algebra structures, 225
- normalization
  - of a simplicial operad in modules, 189
- $n$ -stage for  $E_\infty$  structures, 147
- $\mathcal{O}$ , 158
- obstruction groups
  - in the associative case, 140
  - in the commutative case, 147
  - in the Goerss-Hopkins approach, 115
- obstruction theory, 4
  - in the  $A_\infty$  case, 135
  - in the associative case, 138
  - via  $AQ$ -cohomology of simplicial  $E_\infty$ -algebras
    - in  $E_*$ -modules, 182
    - in  $E_*E$ -comodules, 180
  - via TAQ, 118

- via Gamma cohomology, 121, 133, 145
- via topological derivations, 213
- operad, 20
  - $\mathcal{B}$ , 146
- operad resolution, 163
- operadic derivations, 176
- operads
  - in simplicial sets, 158
- orientation
  - Atiyah-Bott-Shapiro, 5, 106
- orthogonal spectra, 3, 156
- $\mathcal{P}$ -cofibration, 168
- $\mathcal{P}(E)$ , 167
- $\mathcal{P}$ -epi, 167
- $\mathcal{P}$ -equivalence, 168
- perfect universal coefficient formula, 136
- permutative categories, 4, 19, 20
- $\mathcal{P}$ -fibration, 168
- $\pi_*^{st}$ , 121
- $\pi_*^T$ , 122
- pointed simplicial algebraic theory, 122
- Postnikov section
  - of  $E_*$ -algebras in  $E_*E$ -comodules, 186
- Postnikov towers
  - for connective  $R$ -algebras, 215
  - commutative case, 216
- potential  $n$ -stages for  $E_*\mathcal{F}$ -algebras, 183
- $\mathcal{P}$ -projective, 167
- $\mathcal{P}$ -projective cofibration, 167
- progenerator, 35
- real cobordism, 70
- realization space
  - of  $A_\infty$  structures, 152
  - of  $E_\infty$  structures, 152
- Reedy model category
  - of simplicial  $T$ -algebras, 165
  - on simplicial operads, 162
- relative topological derivations, 212
- $S$ -algebra, 201
- $S$ -coalgebras, 27
- simplicial  $T$ -algebra of type  $B_A$ , 184
- small generators, 34
  - in a triangulated category, 45
- small objects
  - in a triangulated category, 44
  - in an abelian category, 34
  - of a stable model category, 59
- smallness in  $\mathcal{D}(R)$ , 45
- smash product, 8, 11, 30, 65, 203
  - external, 9
- $S$ -modules, 3, 15, 16, 87, 133, 156, 201
- smoothness, 119
- spectral model categories, 72
- spectral Quillen pair, 73
- spiral exact sequence, 170
- square-zero extensions, 176, 206, 209
- $\mathcal{S}_{\mathcal{R}}$ , 190
- stabilization
  - and modules of differentials, 218
  - and topological derivations, 219
  - of functors, 217
- stable equivalences of Morita type, 60
- stable equivalences of symmetric spectra, 63
- stable homotopy
  - of  $\Gamma$ -modules, 115, 121, 125, 126
  - of  $\mathcal{T}$ -algebras, 125
  - of algebraic theories, 115, 122, 125
- stable homotopy category, 7, 9, 11, 61
- stable model categories, 59
- Stasheff operad, 135
- strict morphisms, 21
- strict units
  - in the associative case, 138
  - existence of, 140
  - in the commutative case, 148
- suspension spectra, 17, 63
  - of  $\mathcal{T}$ -algebras, 123
- symmetric module spectra, 67
- symmetric monoidal
  - category of spectra, 15
  - lax  $G$ -functor, 90
  - strong  $G$ -functor, 90
  - topological  $G$ -category, 90
- symmetric ring spectra, 58, 67
- symmetric spectra, 3, 15, 24, 58, 62, 87, 156
- $t$ -structures, 50
- TAQ, 117

- $TAQ_S^*(H\mathbb{F}_p, H\mathbb{F}_p)$ , 223
- tilting complexes for rings, 49
- tilting modules, 55
  - examples, 56, 57
- topological André-Quillen cohomology,
  - 5, 202, 207
- topological André-Quillen homology,
  - 115, 117, 126, 207
- topological derivations, 202, 211
  - spectrum of, 211
- topological Hochschild cohomology, 83,
  - 211
- topological  $K$ -theory, 3, 4, 88, 148
  - equivariant, 97
- topological singular extension, 214
- triangulated categories, 34, 43
- twisted half-smash product, 9
  
- universal deformation, 194
- universe, 8
- unstable algebras over the Dyer-Lashof
  - algebra, 189
- $\mathcal{UR}$ , 190