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Representation Theory of Artin Algebras

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To our parents

Charles and Ida

Ivar and Alma

Olaf and Svanhild

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Frontmatter
More information

Contents

Introduction

A major concern of elementary linear algebra is the description of how one linear transformation can act on a finite dimensional vector space over a field. Stated in simplest terms, the central problem of this book is to describe how a finite number of linear transformations can act simultaneously on a finite dimensional vector space. While the language of linear algebra suffices in dealing with one transformation on a vector space, it is inadequate to the more general task of dealing with several linear transformations acting simultaneously. As so often happens, when straightforward approaches to seemingly simple problems fail, one adopts a more devious strategy, usually involving a more abstract approach to the problem. In our case, this more abstract approach is called the representation theory of finite dimensional algebras, which in its broadest terms is the study of modules over finite dimensional algebras. One of the advantages of the module theoretic approach is that the language and machinery of both category theory and homological algebra become available. While these theories play a central role in this book, no extensive knowledge of these subjects is required, since only the most elementary concepts and results, as contained in most introductory courses or books on homological algebra, are assumed.

Although many of the concepts and results presented here are of recent origin, having been developed for the most part over the past twenty-five years, the subject itself dates from the middle part of the nineteenth century with the discovery of the quaternions, the first noncommutative field, and the subsequent development, during the first part of this century, of the theory of semisimple finite dimensional algebras over fields. This well developed theory has played an important role in such classical subjects as the representation theory of finite groups over the complex numbers and the Brauer groups of fields, the first of which has proven to be a powerful tool in finite group theory and algebraic number theory, while the second is an object of deep significance also in algebraic number theory as well as in algebraic geometry and abstract field theory. However, since the theory of semisimple finite dimensional algebras is not only well developed but also easily accessible either through elementary algebra courses or textbooks, we assume the reader is familiar with this theory which we use freely in discussing non-semisimple finite dimensional algebras, the algebras of primary concern to us in this book.

While the interest in nonsemisimple finite dimensional algebras goes back to the latter part of the nineteenth century, the development of a general theory of these algebras has been much slower and more sporadic than the semisimple theory. Until recently much of the work has been concentrated on studying specific types of algebras such as modular group algebras, the Kronecker algebra and Nakayama algebras, to name a few. This tradition continues to this day. For instance, algebras of finite representation types have been studied extensively, as have hereditary algebras of finite and tame representation type. But there is at least one respect in which the more recent work differs sharply from the earlier work. There is now a much more highly developed theoretical framework, which has made a more systematic, less *ad hoc* approach to the subject possible. It is our purpose in this book to give an introduction to the part of the theory built around almost split sequences. While this necessitates discussing other aspects of the general theory such as categories of modules modulo various subcategories and the dual of the transpose, other important topics such as coverings, tilting, bocses, vector space categories, posets, derived categories, homologically finite subcategories and finitely presented functors are not dealt with. We do not discuss tame algebras, except for one example, and we do not deal with quantum groups, perverse sheaves or quasihereditary algebras. Some of the topics which are omitted are basic to representation theory, and our original plan, and even first manuscript, included many of them. Since we wanted to include enough preliminary material to make the book accessible to graduate students, space requirements made it necessary to modify our original ambition and leave out many developments, including some of our own favorite ones, from this volume. In particular we postponed the treatment of finitely presented functors since we felt there would not be enough space in the present volume to illustrate their use. It is hoped that there will be a forthcoming volume dealing with other aspects of the subject. Also some other aspects are dealt with in the books [GaRo], [Hap2], [JL], [Pr], [Rin3], [Si].

Besides personal taste, our reason for concentrating on the theory centered around almost split sequences is that these invariants of indecomposable modules appear either explicitly or implicitly in much of the recent work on the subject. We illustrate this point by giving applications to Grothendieck groups, criteria for finite representation type, hereditary algebras of finite representation type and the Kronecker algebra, which is of tame but not finite representation type.

Our proof of the existence theorem for almost split sequences has not appeared before in the literature. It is based on an easily derived, but remarkably useful, relationship between the dimensions of vector spaces of homomorphisms between modules. Amongst other things, this formula comes up naturally in studying cycles of morphisms and their impact on the question of when modules are determined by their composition factors, as well as in the theory of morphisms determined by modules, which is in essence a method for classifying homomorphisms. In fact, one of the important features of the present day representation theory of finite dimensional algebras is this concern with morphisms between modules in addition to the modules themselves.

Although we have been pretending that this book is about finite dimensional algebras over fields, it is for the most part concerned with the slightly more general class of rings called artin algebras which are algebras Λ over commutative artin rings R with Λ a finitely generated R-module. The reason for this is that while the added generality considerably widens the applicability of the theory, there is little added complication in developing the theory once one has established the duality theory for finitely generated modules over commutative artin rings. Of course, we have not hesitated to specialize to fields or algebraically closed fields when this is necessary or convenient.

The book is divided into eleven chapters, each of which is subdivided into sections. The first two chapters contain relevant background material on artin rings and algebras. Chapter III provides a large source of examples of artin algebras and their module categories, especially through the discussion of quivers and their representations. The next four chapters contain basic material centered around almost split sequences and Auslander–Reiten quivers. The first seven chapters together with Chapter VIII on hereditary algebras form the core of the book. The last four chapters are more or less independent of each other. Following each chapter is a set of exercises of various degrees of depth and complexity. Some are superficial "finger exercises" while others are outlines of proofs of significant theories not covered in the text. These exercises are followed by notes containing brief historical and bibliographical comments as well as suggested further readings. There has been no attempt made to give a comprehensive list of references. Many important papers related to the material presented in this book do not appear in our reference list. The reader can consult the books and papers quoted for further references. We provide no historical comments or specific references on standard facts on ring theory and homological algebra, but give references to appropriate textbooks.

At the end of the book we list conjectures and open problems, some of which are well known questions in the area. We give some background and references for what is already known.

Finally we would like to thank various people for making helpful comments on parts of the book, especially Dieter Happel and Svein Arne Sikko, and also Øyvind Bakke, Bill Crawley-Boevey, Wei Du, Otto Kerner, Henning Krause, Shiping Liu, Brit Rohnes, Claus Michael Ringel, Øyvind Solberg, Gordana Todorov, Stig Venås and Dan Zacharia. In addition we are grateful to students at Brandeis, Düsseldorf, Syracuse and Trondheim for trying out various versions of the book.

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Maurice Auslander died on November 18, 1994. We deeply regret that he did not live to see the book in print.